

A scanning electron micrograph (SEM) of a circular photonic crystal cavity. The structure consists of a central ring of holes surrounded by a larger ring of holes, all within a circular substrate. The central ring has a diameter of approximately 1.5 micrometers, and the outer ring has a diameter of approximately 4.5 micrometers. The holes are arranged in a hexagonal lattice. The image is in grayscale, with the central ring appearing as a bright, glowing structure against a darker background.

Squeezing with parametric modulation close to quantum regime

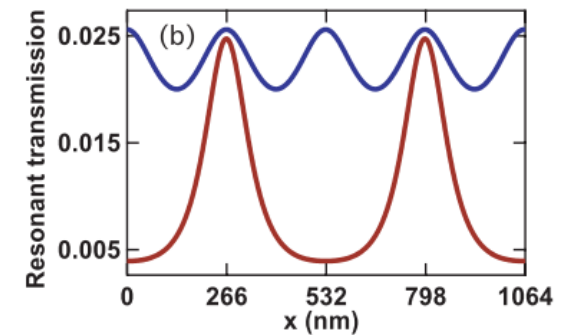
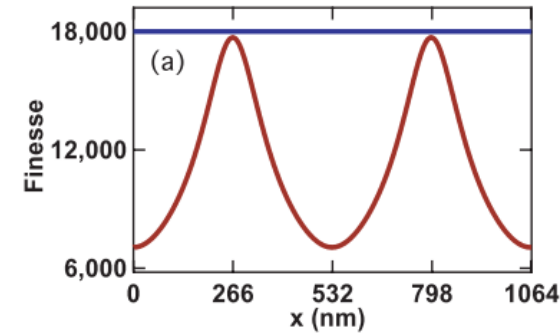
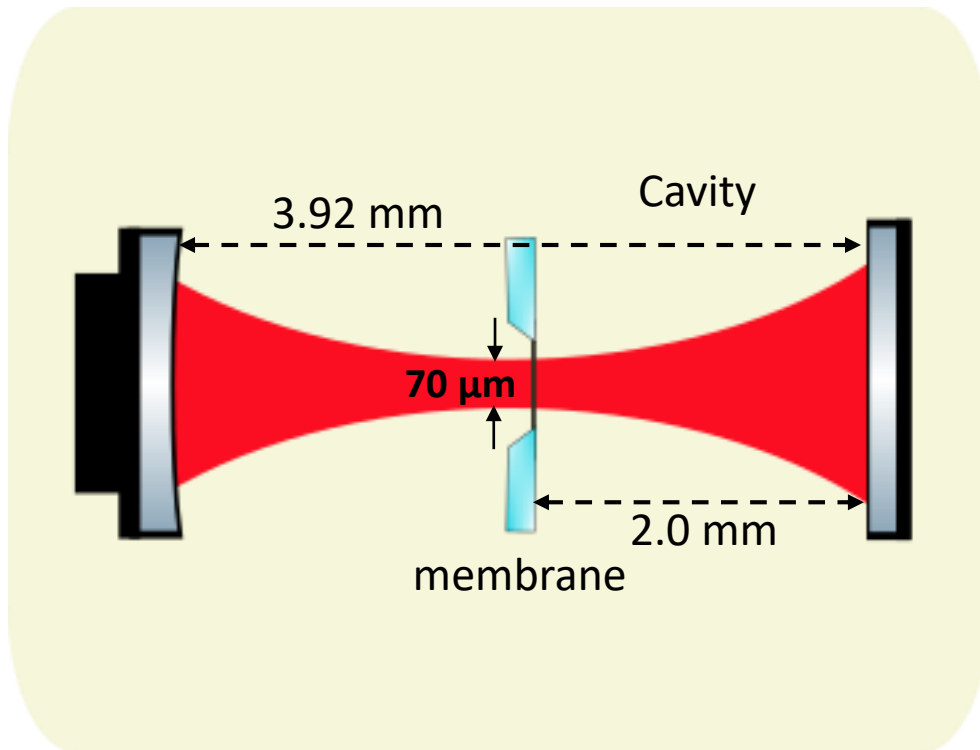
Avishek Chowdhury

Paolo Vezio

Francesco Marino

Francesco Marin

Cavity opto-mechanics: membrane in the middle

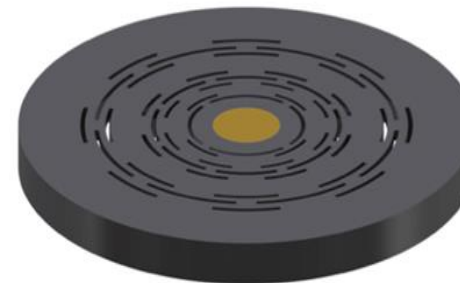


Jayich et al. *NJP* 10 (2008) 095008

Tunable finesse as a function of the membrane position

Cavity:

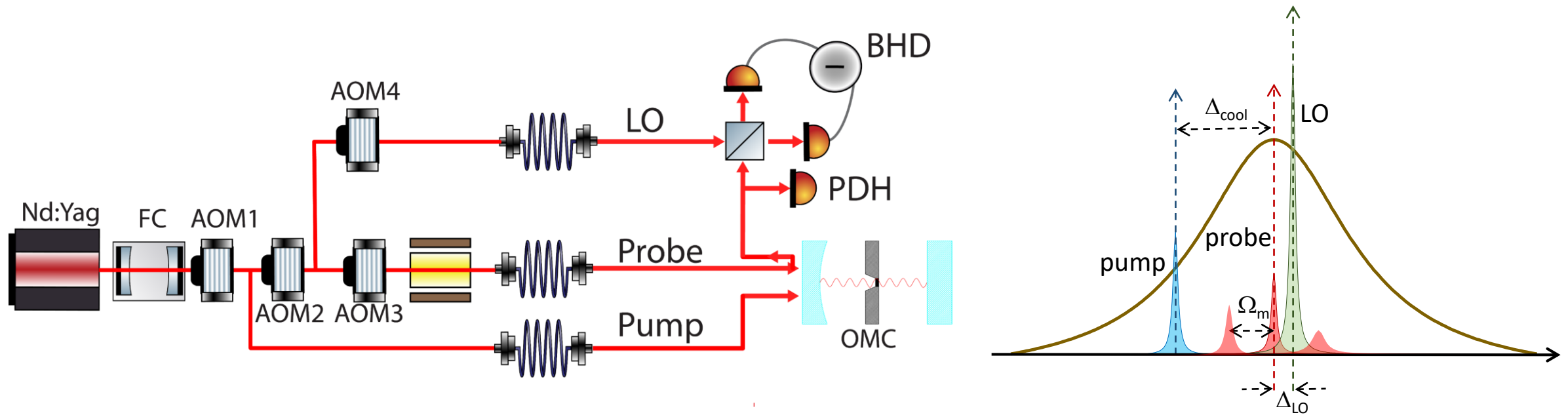
- Finesse $\sim 20,000$
- Cavity linewidth ~ 1.9 MHz



Membrane:

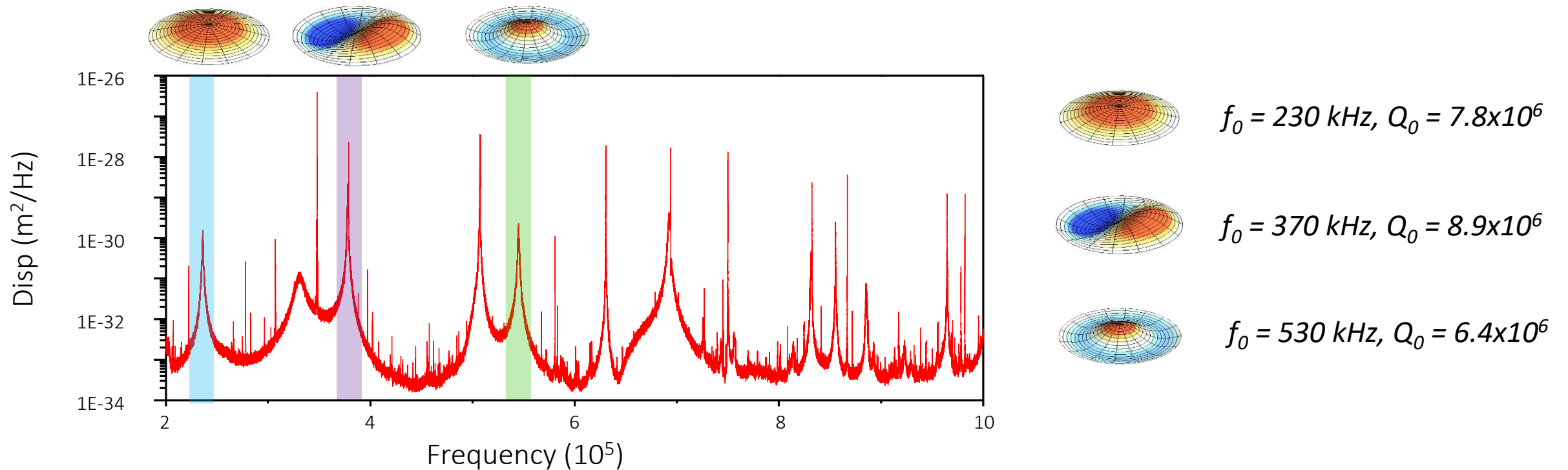
- Thickness ~ 100 nm
- Diameter ~ 1.64 mm
- $Q_{\text{cryogenic temp}} \sim 10^7$

Simplified optical set-up



- **Weak probe:** to detect the displacement of the mechanical mode, in resonance with the cavity.
- **Strong pump:** detuned from the cavity, used to optically cool down the mechanical mode.
- **LO beam:** Local Oscillator beam used to make heterodyne measurement on the mechanical mode.

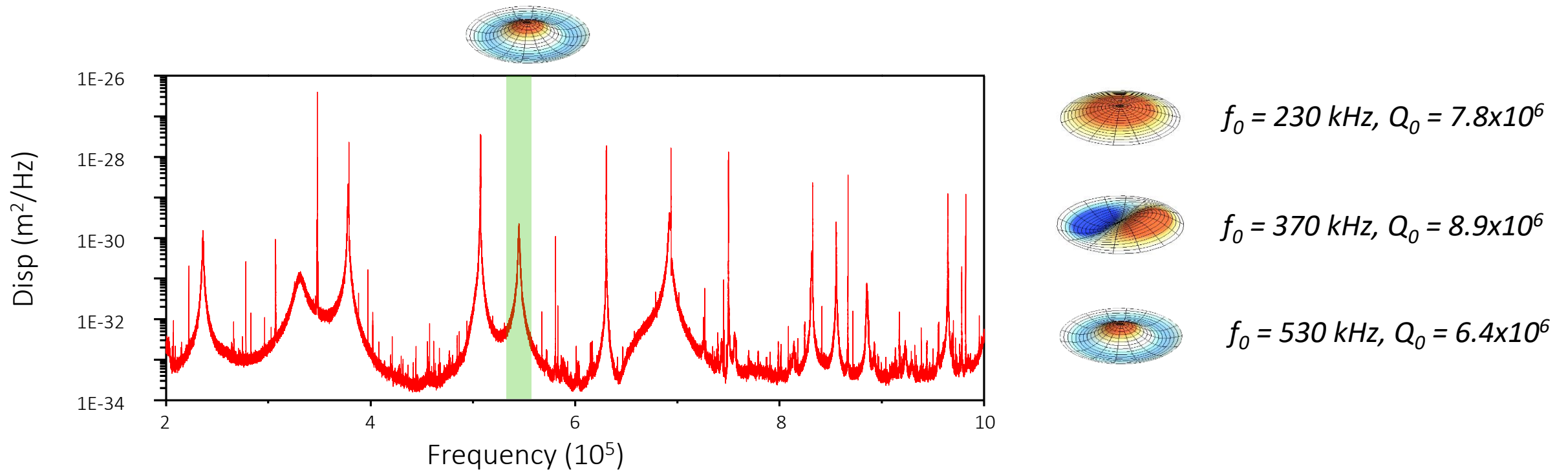
Membrane modes: eigen-modes



Choice of the modes

- Considerably high Q factor.
- High optomechanical coupling: highly dependent on the effective overlap between the laser beam and the membrane eigen mode.

Membrane modes: eigen-modes



We choose: Mode at 530 kHz

- Q factor: 6.4×10^6 .
- With an optomechanical coupling $g_0 \sim 5 \text{ Hz}$.

Cooling a mechanical mode

$$\delta\Omega_m(\omega) = g^2 \frac{\Omega_m}{\omega} \left[\frac{\Delta + \omega}{(\Delta + \omega)^2 + \kappa^2/4} + \frac{\Delta - \omega}{(\Delta - \omega)^2 + \kappa^2/4} \right]$$

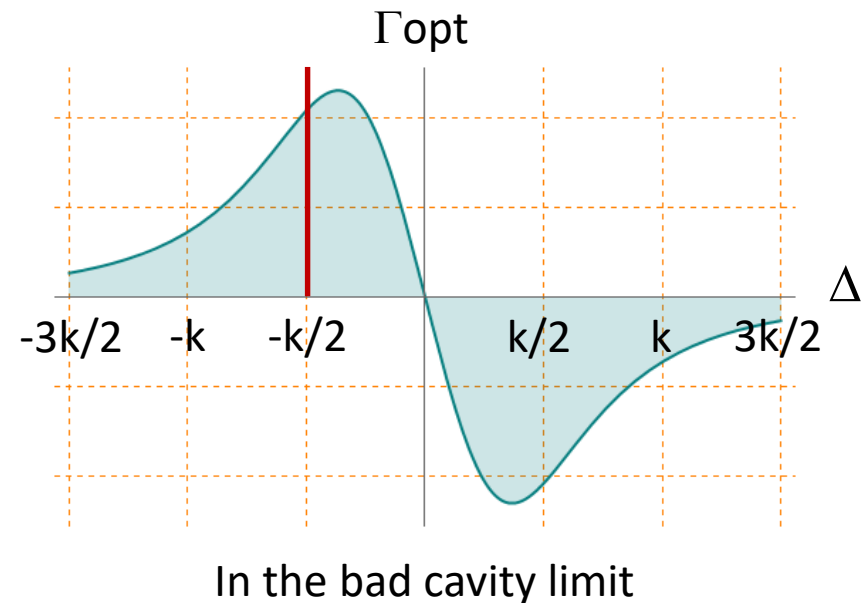
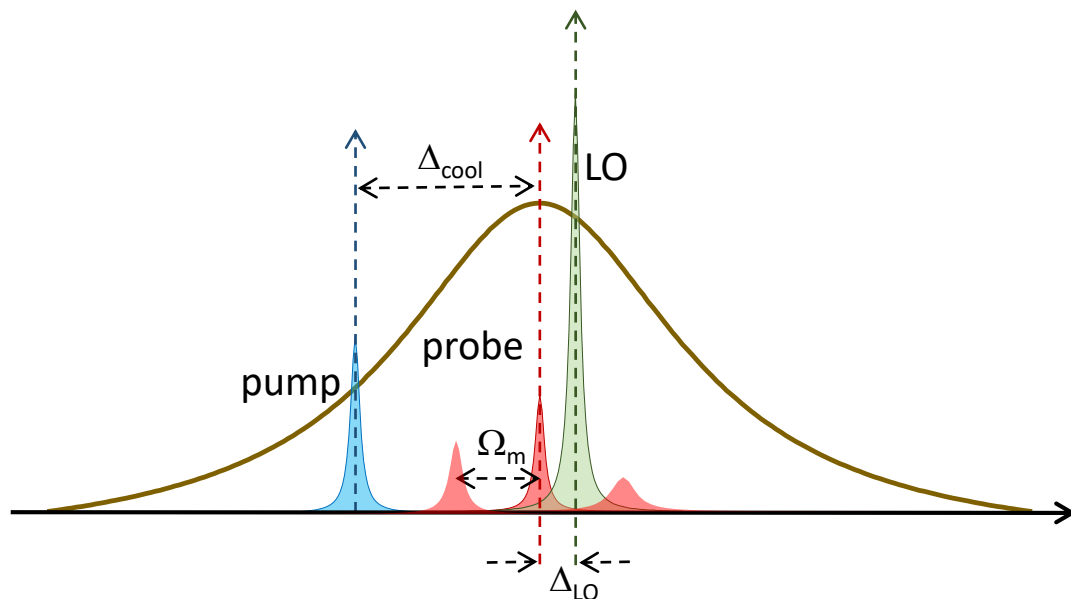
Optical Spring effect

$$\Gamma_{\text{opt}}(\omega) = g^2 \frac{\Omega_m}{\omega} \left[\frac{\kappa}{(\Delta + \omega)^2 + \kappa^2/4} - \frac{\kappa}{(\Delta - \omega)^2 + \kappa^2/4} \right]$$

Damping of the mode 'm'

$$g^2 = g_0^2 \bar{n}_{\text{cav}}$$

$$\Gamma_{\text{opt}} \sim f(n_{\text{cav}}, \Delta)$$



Cooling a mechanical mode

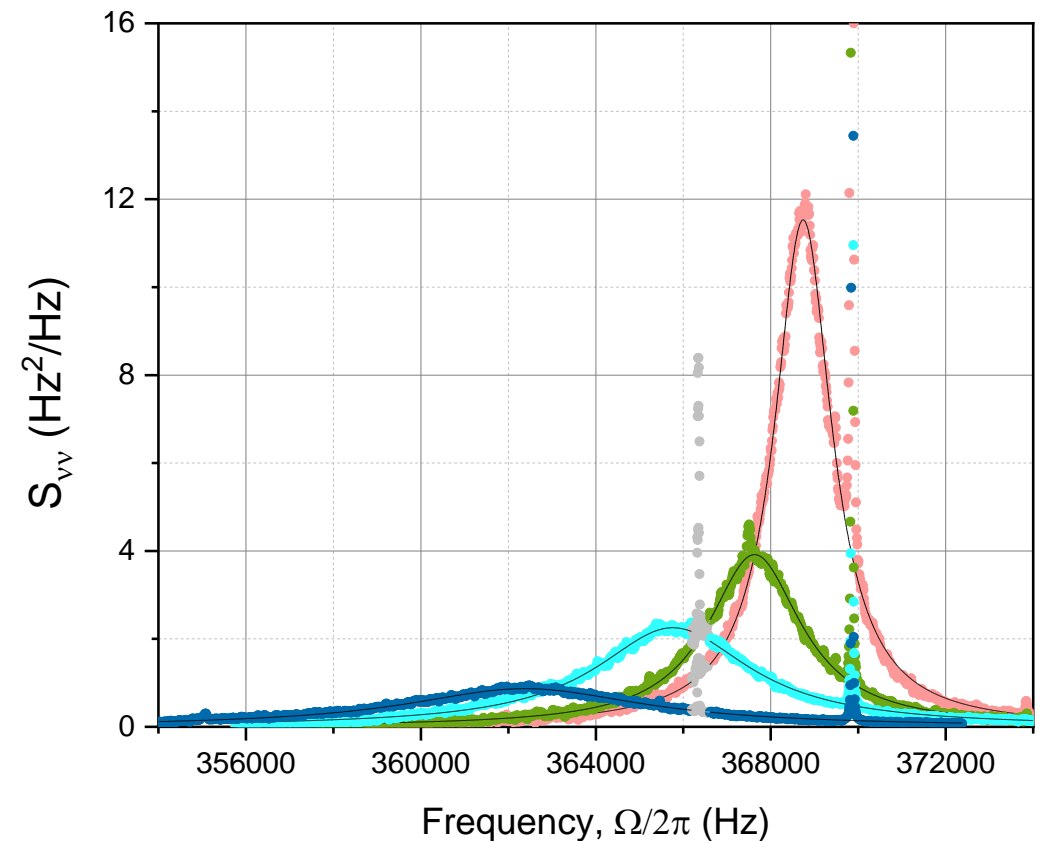
$$\Gamma_{\text{opt}}(\omega) = g^2 \frac{\Omega_{\text{m}}}{\omega} \left[\frac{\kappa}{(\Delta + \omega)^2 + \kappa^2/4} - \frac{\kappa}{(\Delta - \omega)^2 + \kappa^2/4} \right]$$

$$\Delta \sim \kappa/2 = 1 \text{ MHz} \quad g^2 = g_0^2 \bar{n}_{\text{cav}}$$

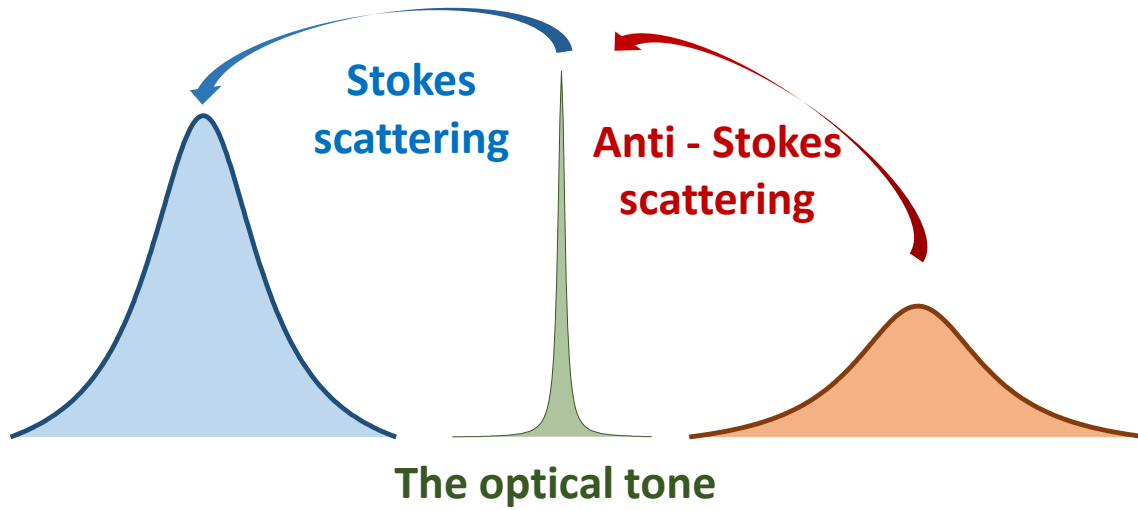
Cooling measurement relying on the measurement and calibration of motion induced scattering of light giving an average phonon occupancy $\langle n \rangle$

Quantum thermometry

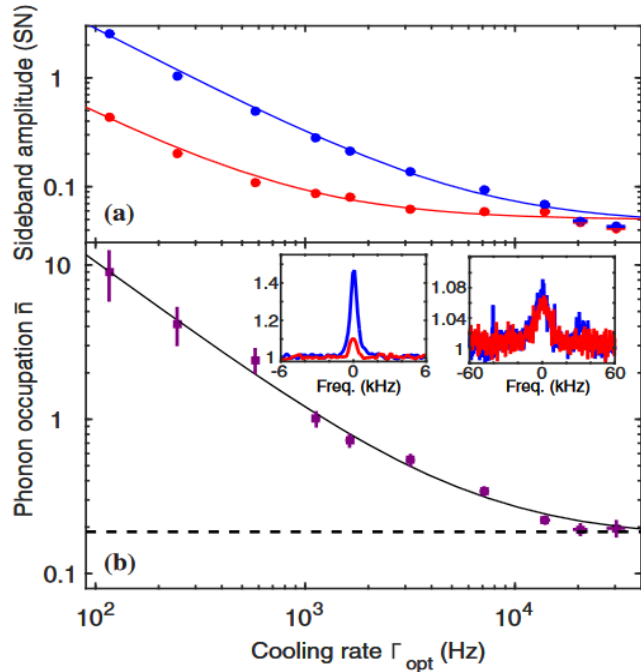
- Ratio of Stokes and anti-Stokes sideband from the mechanical oscillator: $\frac{\langle n \rangle + 1}{\langle n \rangle}$



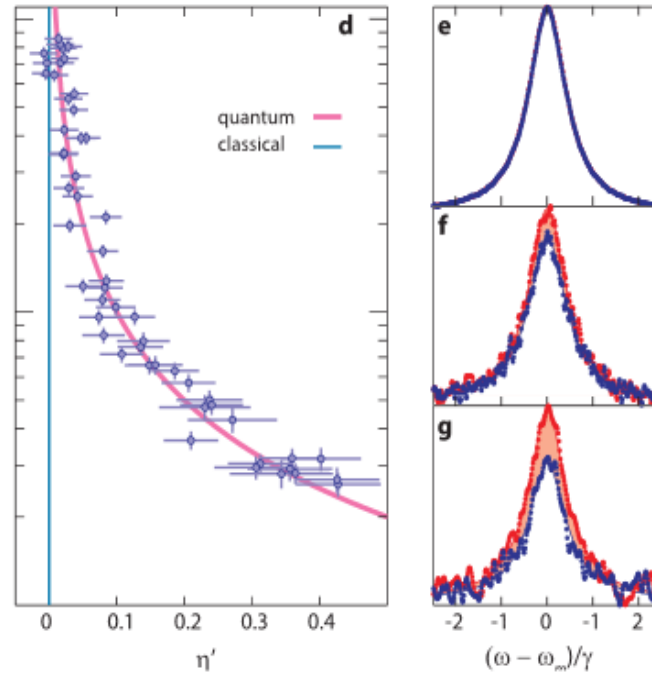
The scattering picture



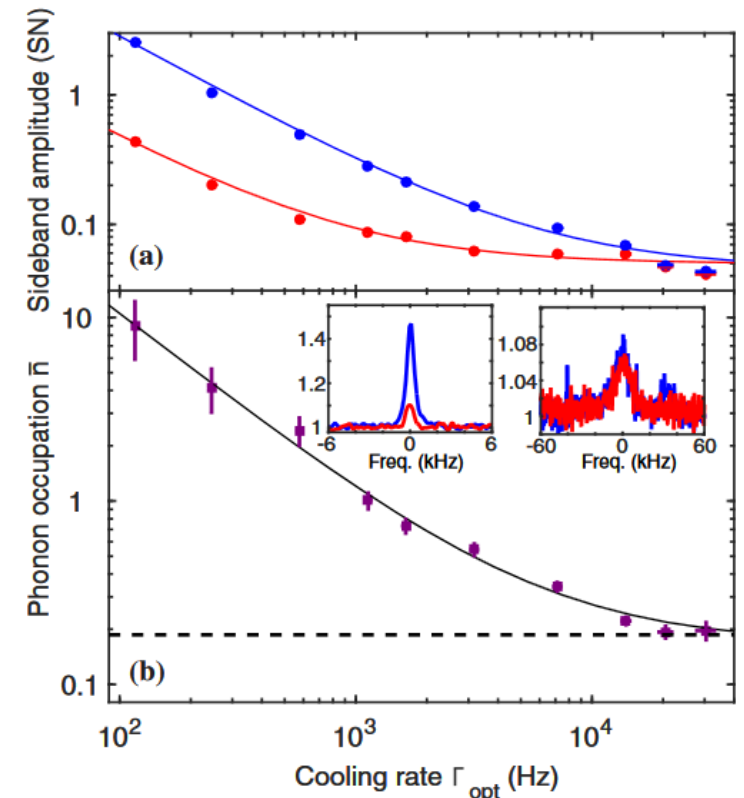
$$\text{Ratio} = \frac{\text{Stokes rate}}{\text{anti Stokes rate}} = \frac{\langle n \rangle + 1}{\langle n \rangle}$$



Regal, PRL 116, 063601 (2016)

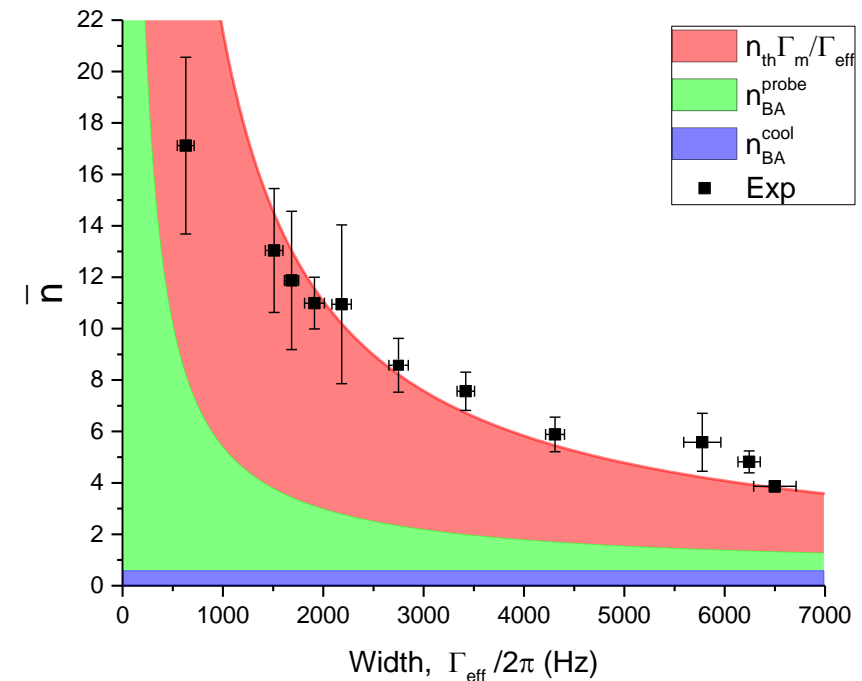
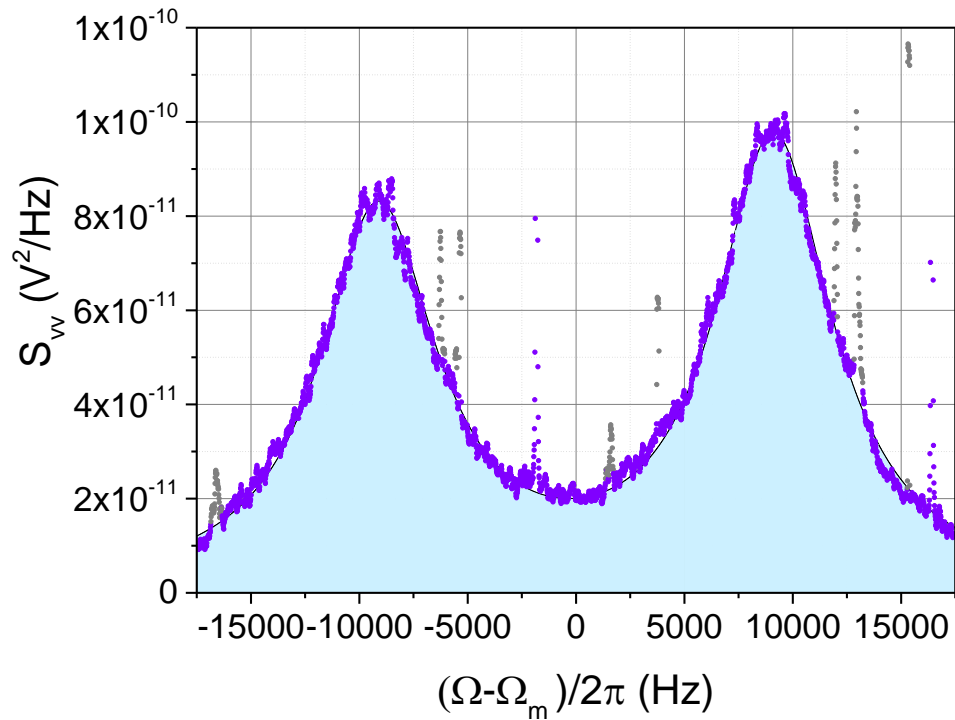
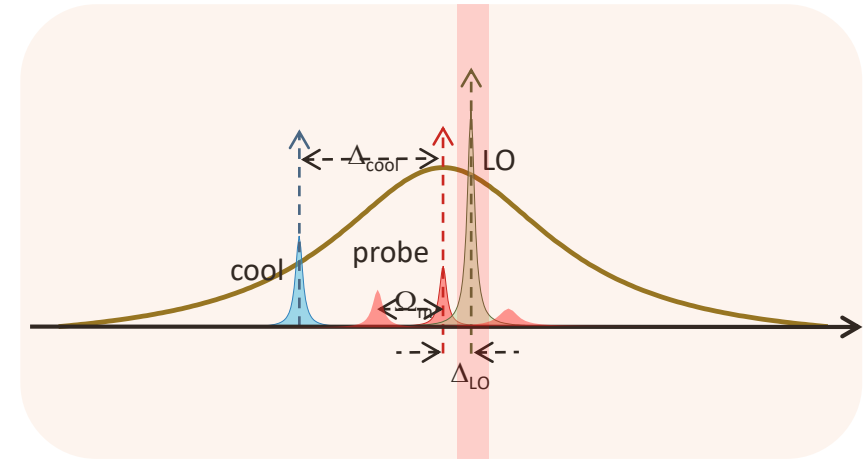
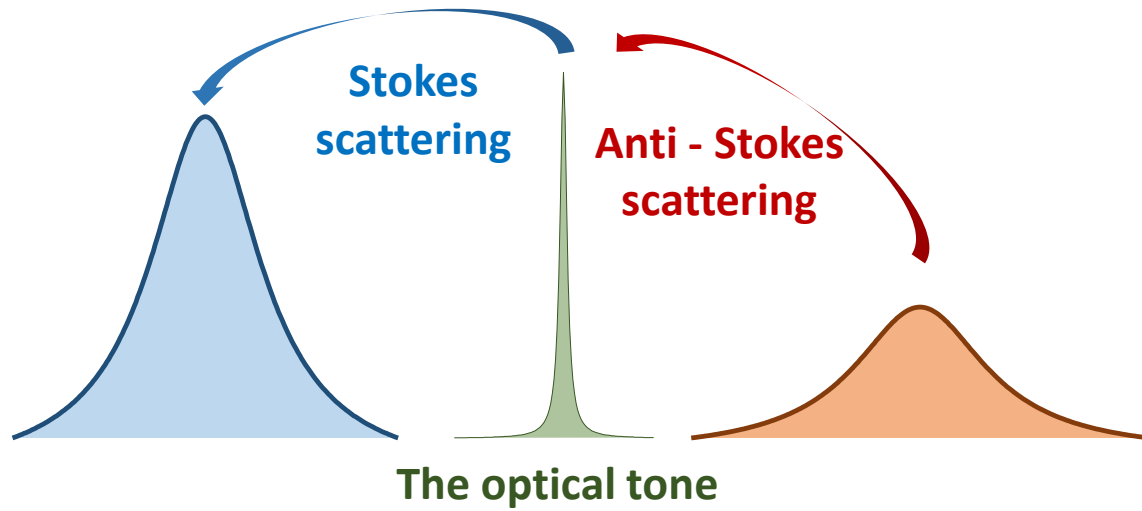


Painter, PRL 108, 033602 (2012)



Regal, PRL 116, 063601 (2016)

The scattering picture



Towards zero-point state

Lower bound to the position of SHO:

$$\Delta X \Delta Y \geq \frac{h}{2m_{eff}\Omega_m} = x_{ZPF}$$

Typical squeezing measurement: Measurement of noise in one quadrature -> Increase of noise in the other.

Already existing squeezing schemes:

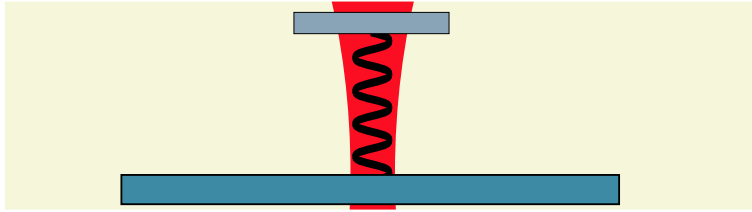
- Pulsed laser cooling of oscillator: ***requires short pulsed laser.***
- Squeezing via backaction evading measurements (BAE): ***requires motional state close to the quantum regime.***

Parametric squeezing

- No strict requirement on the starting occupation number of motional state.
- Experiments can be done in the 'bad-cavity' regime.

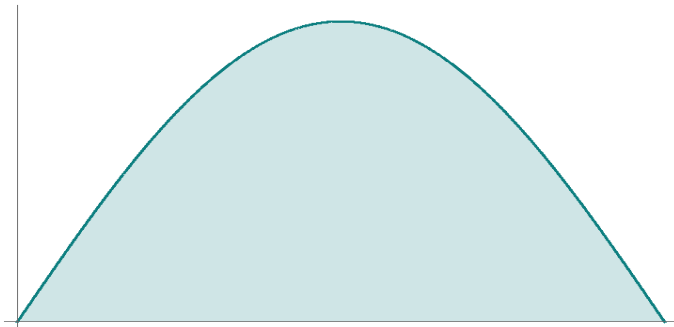
Parametric cooling of an oscillator

Modulation of spring constant at twice the resonant frequency

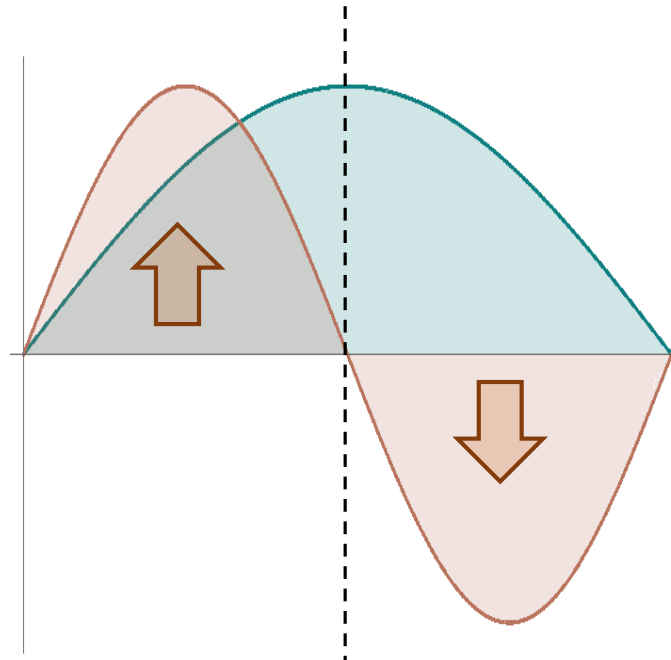


$$k(t) = k_0 + k_p(t)$$

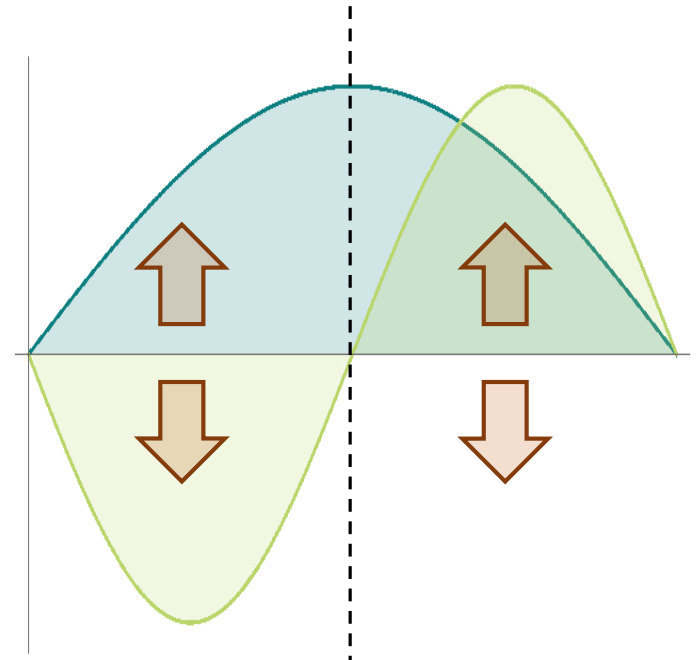
$k_0 \rightarrow$ intrinsic spring constant
 $k_p \sim \text{Cos}(2\pi f_0 t) \rightarrow$ parametric modulation



Motion of a harmonic oscillator



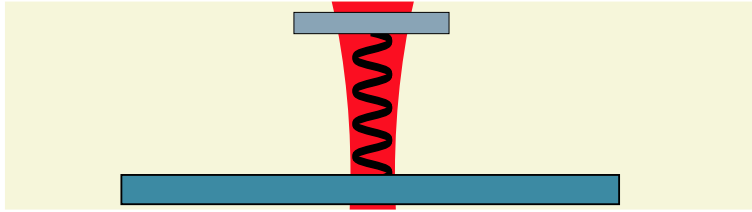
Parametric excitation @ $2f_0$



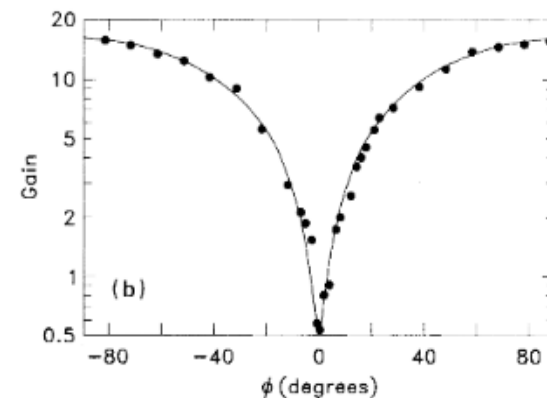
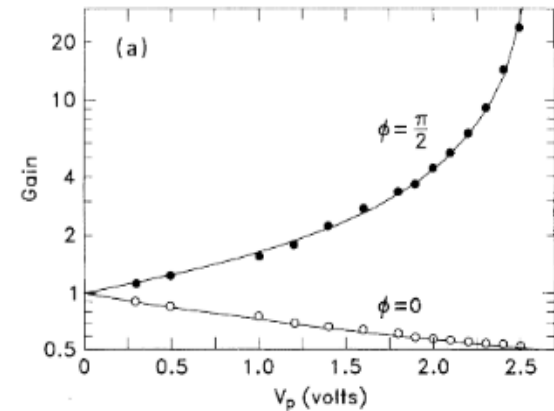
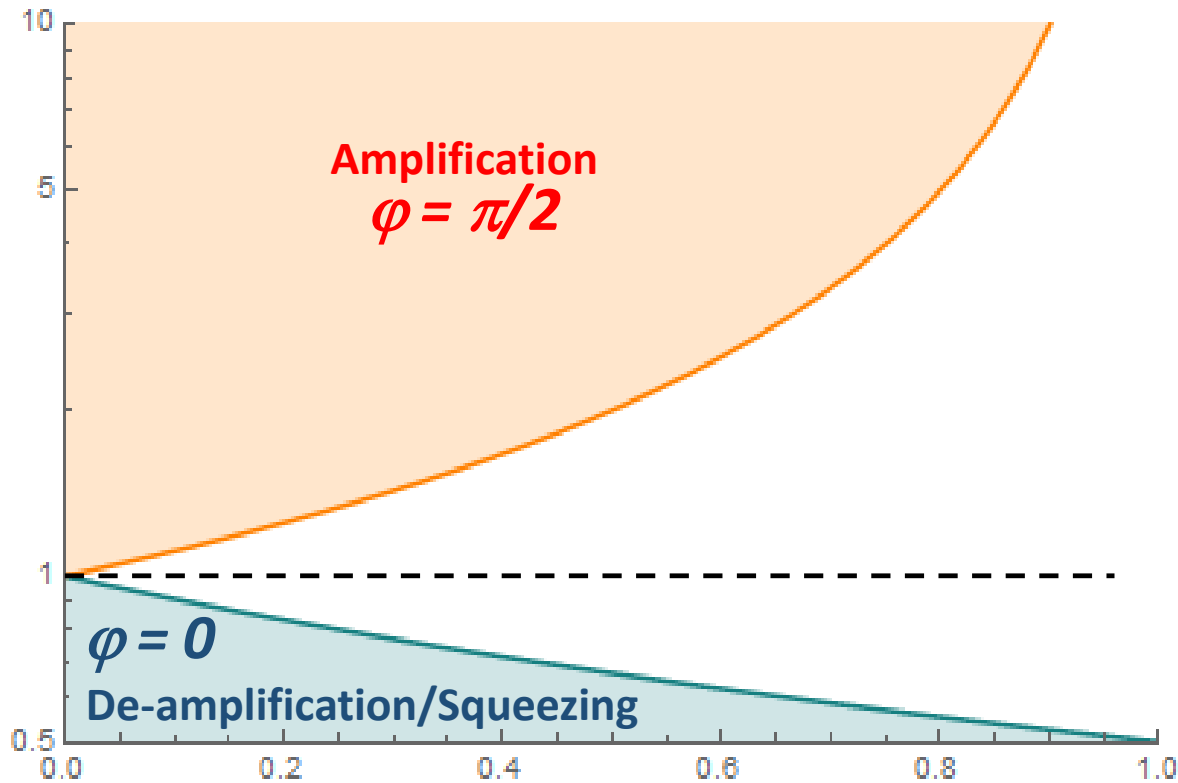
Parametric excitation @ $2f_0 + \pi$

Parametric cooling of an oscillator

Modulation of spring constant at twice the resonant frequency



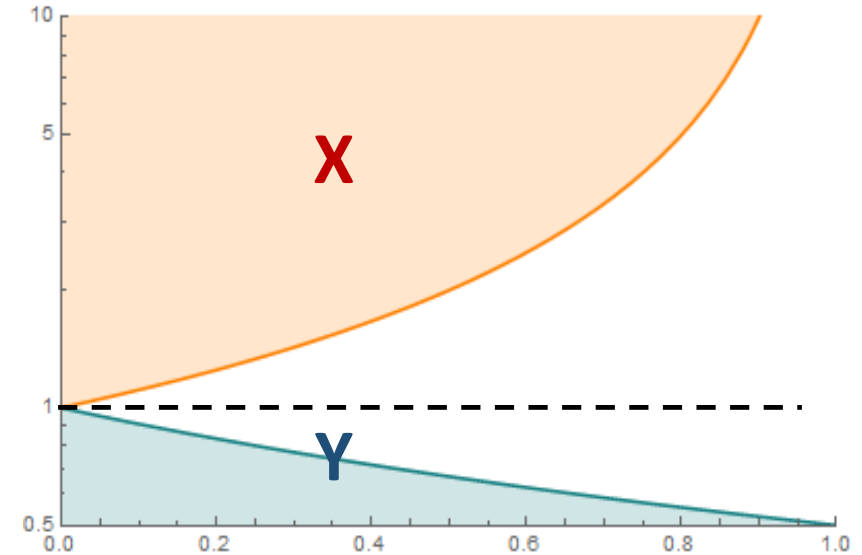
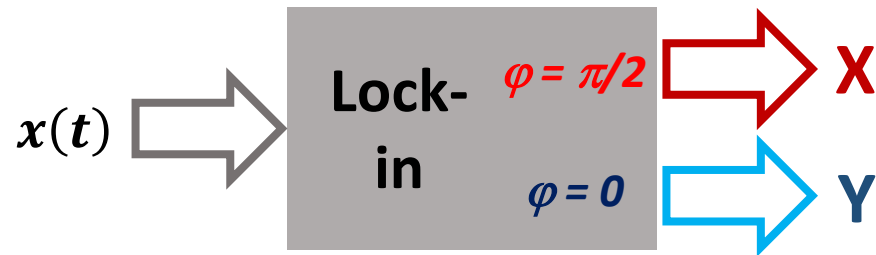
$$s(\varphi) = \left[\frac{\cos^2(\varphi)}{(1 + P_{norm})^2} + \frac{\sin^2(\varphi)}{(1 - P_{norm})^2} \right]^{1/2}$$



Rugar et al. PRL 67, 6 (1991)

Parametric cooling of an oscillator

$$x(t) = X \cos(2\pi f_0 t) + Y \sin(2\pi f_0 t)$$



Squeezing the displacement in one quadrature while adding noise on the other:
Leads to thermal squeezing of a mechanical oscillator

Parametric cooling of an oscillator

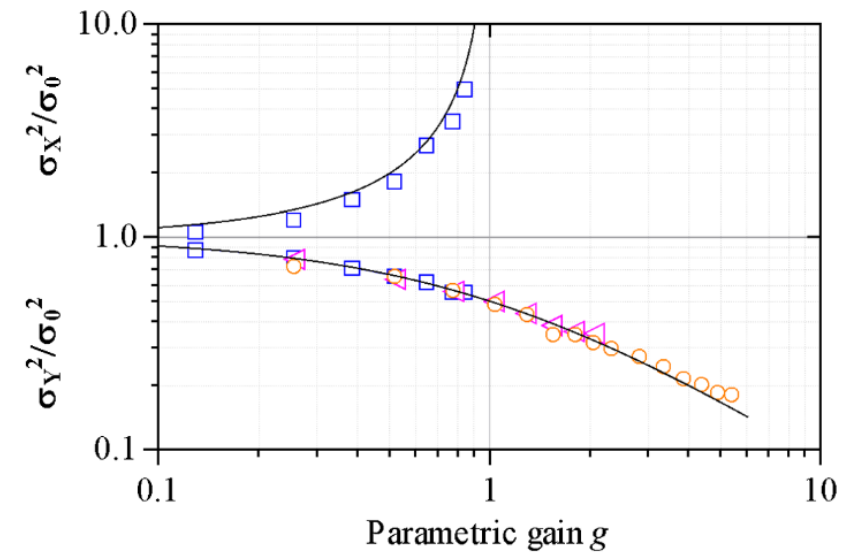
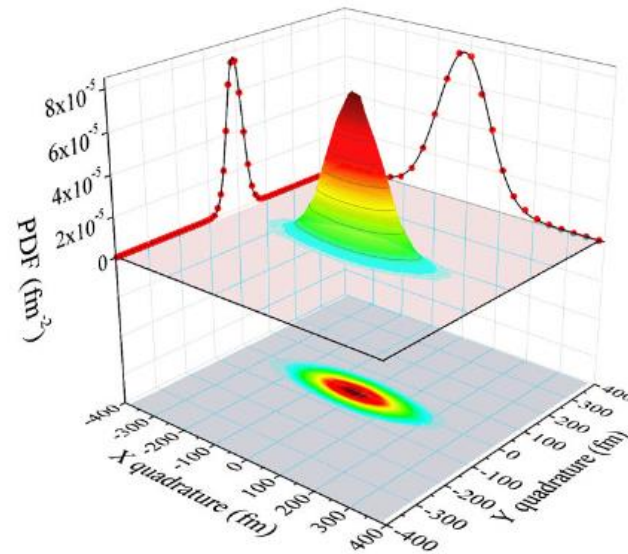
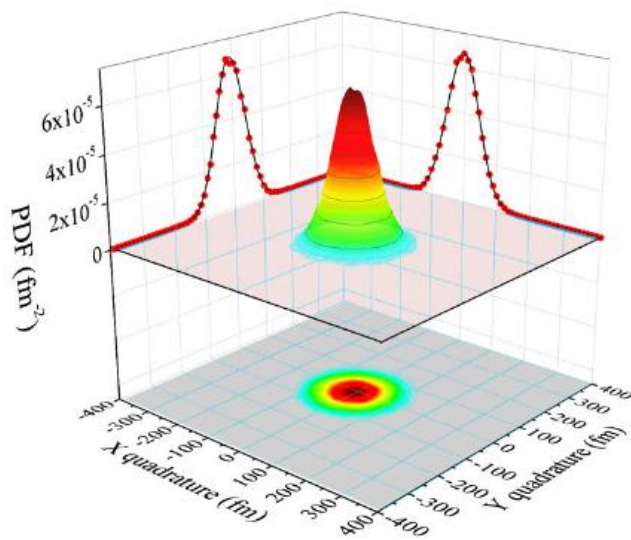
PRL 112, 023601 (2014)

PHYSICAL REVIEW LETTERS

week ending
17 JANUARY 2014

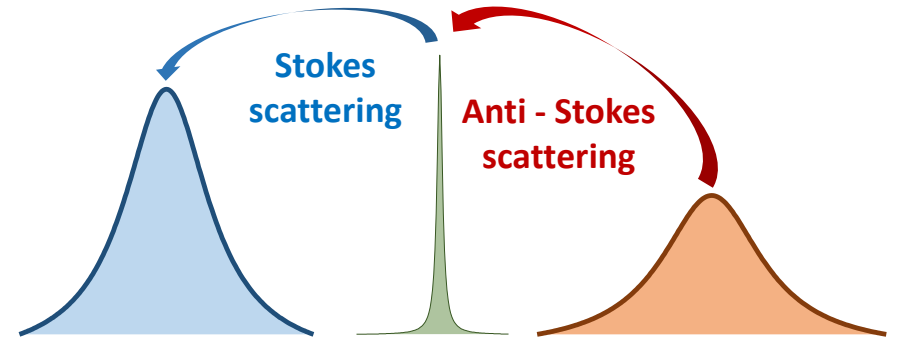
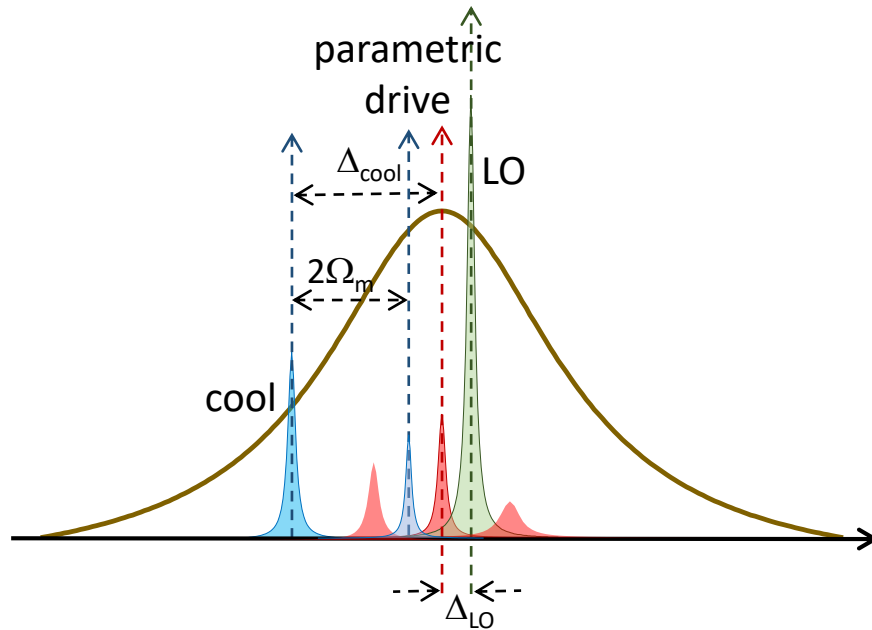
Squeezing a Thermal Mechanical Oscillator by Stabilized Parametric Effect on the Optical Spring

A. Pontin,^{1,2} M. Bonaldi,^{3,4} A. Borrielli,^{3,4} F. S. Cataliotti,^{5,6,7} F. Marino,^{7,8} G. A. Prodi,^{1,2} E. Serra,^{1,9} and F. Marin^{5,6,7,*}



Squeezed thermal state

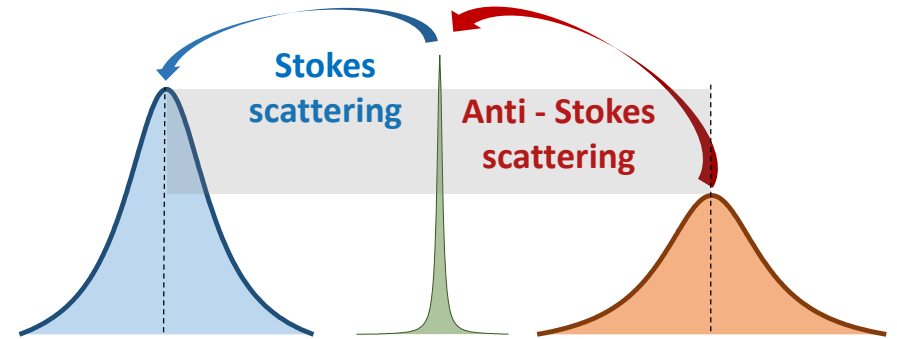
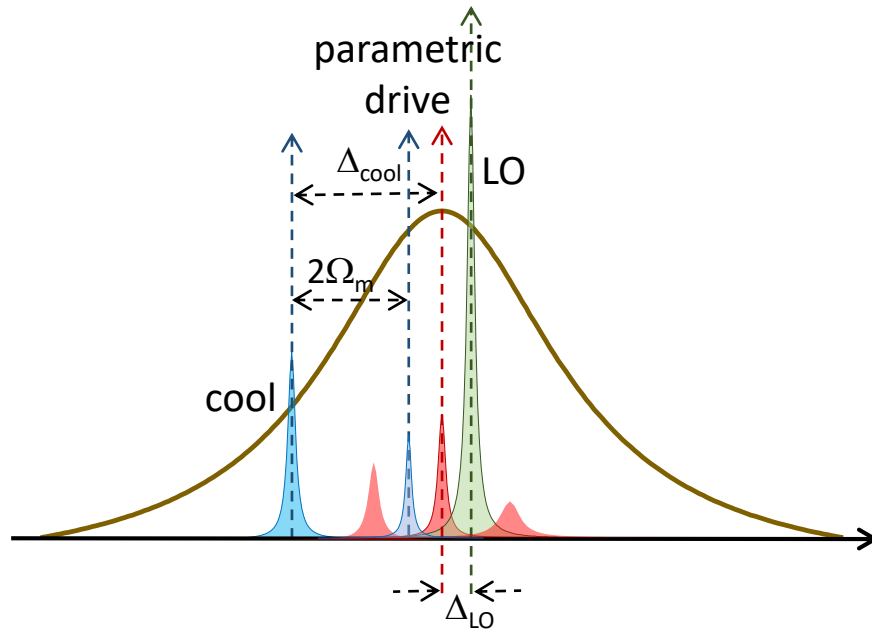
Parametric cooling close to the quantum regime



Weak parametric tone (@ $2\Omega_m$) added to the pump beam

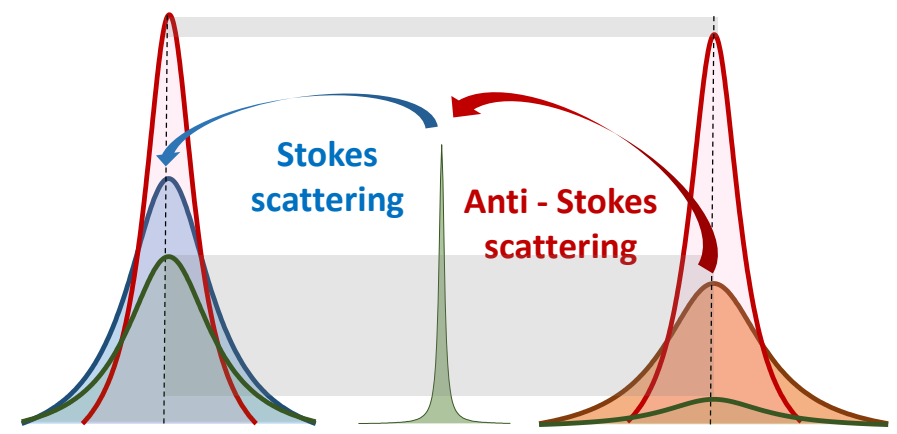
$$I_{pump} = I_{cool} + I_{par}$$

Parametric cooling close to the quantum regime

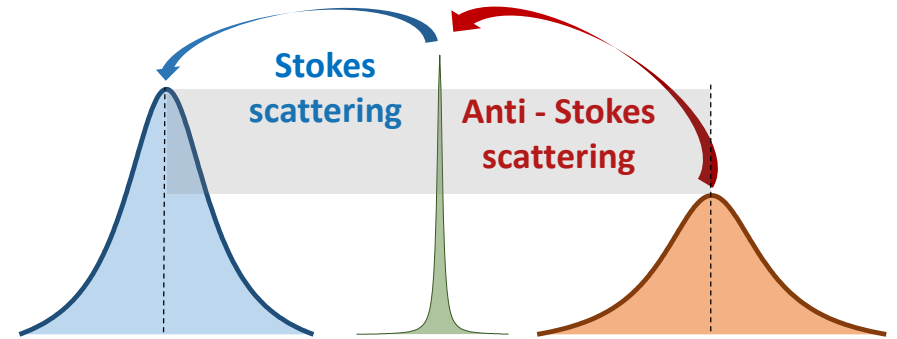
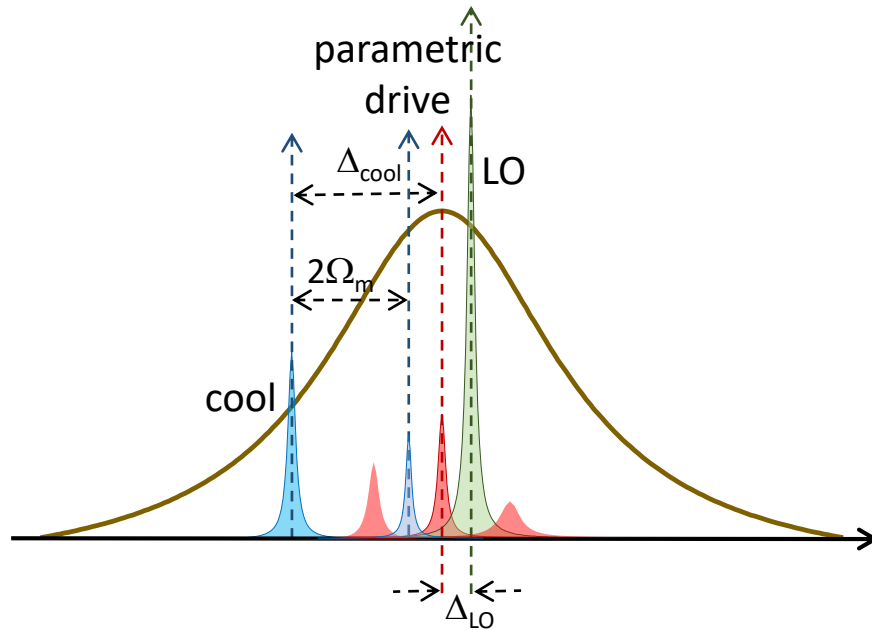


$$S_{anti-Stokes} = \frac{\Gamma_{opt}}{2} \left[\frac{n + s/2}{\omega^2 + \left(\frac{\Gamma_-}{2}\right)^2} + \frac{n - s/2}{\omega^2 + \left(\frac{\Gamma_+}{2}\right)^2} \right]$$

$$S_{Stokes} = \frac{\Gamma_{opt}}{2} \left[\frac{n + 1 - s/2}{\omega^2 + \left(\frac{\Gamma_-}{2}\right)^2} + \frac{n + 1 + s/2}{\omega^2 + \left(\frac{\Gamma_+}{2}\right)^2} \right]$$

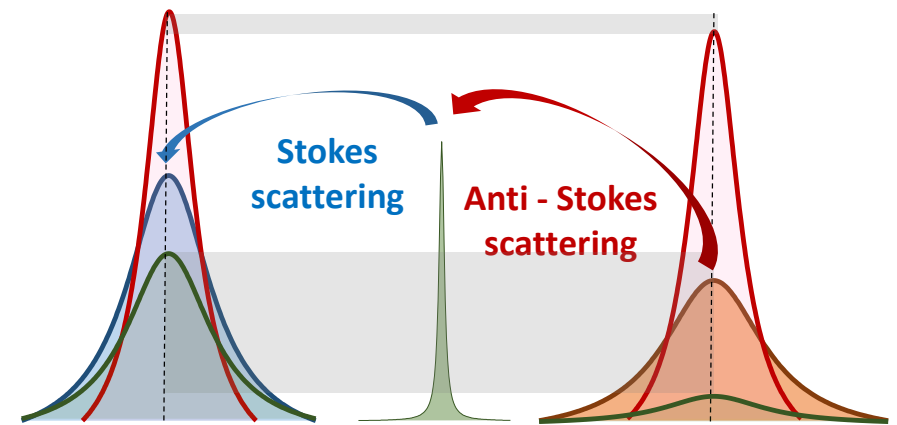
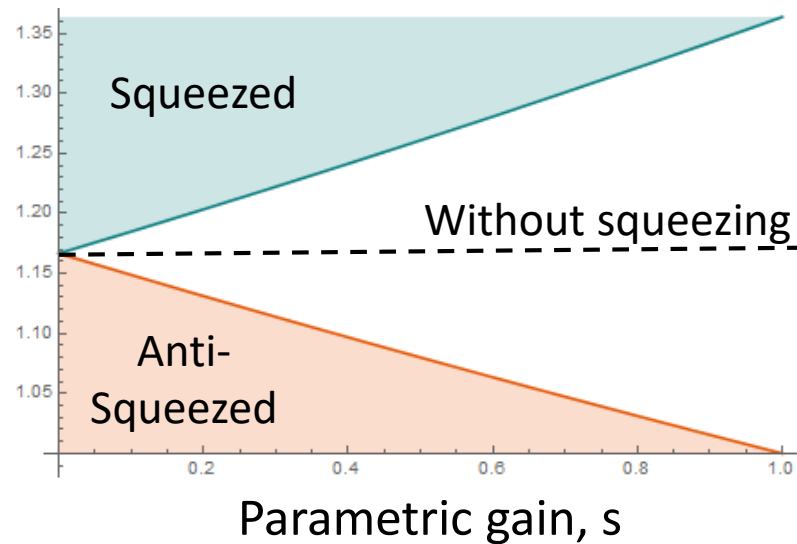


Parametric cooling close to the quantum regime

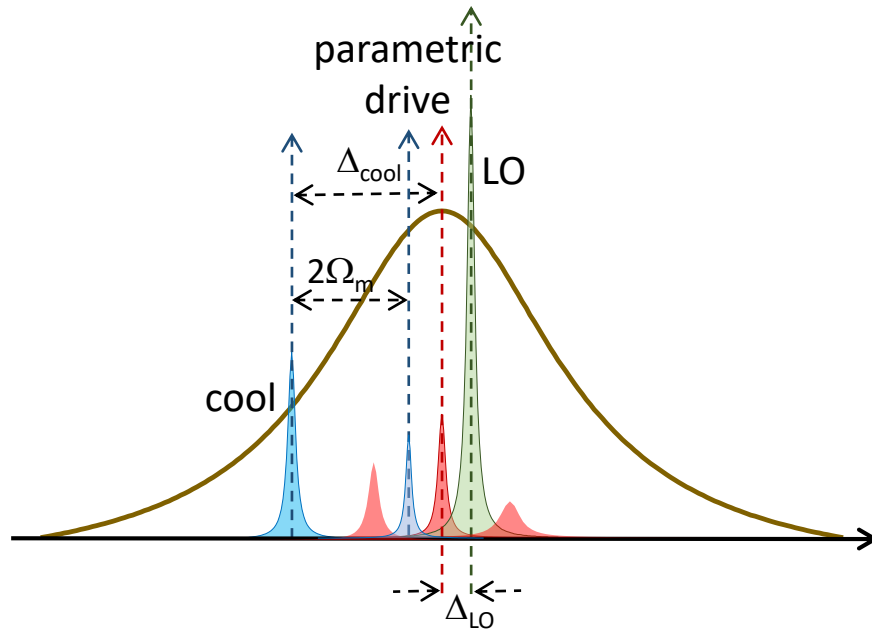


SB ratio, $R_Y : \frac{n+1+s/2}{n-s/2}$

SB ratio, $R_X : \frac{n+1-s/2}{n+s/2}$

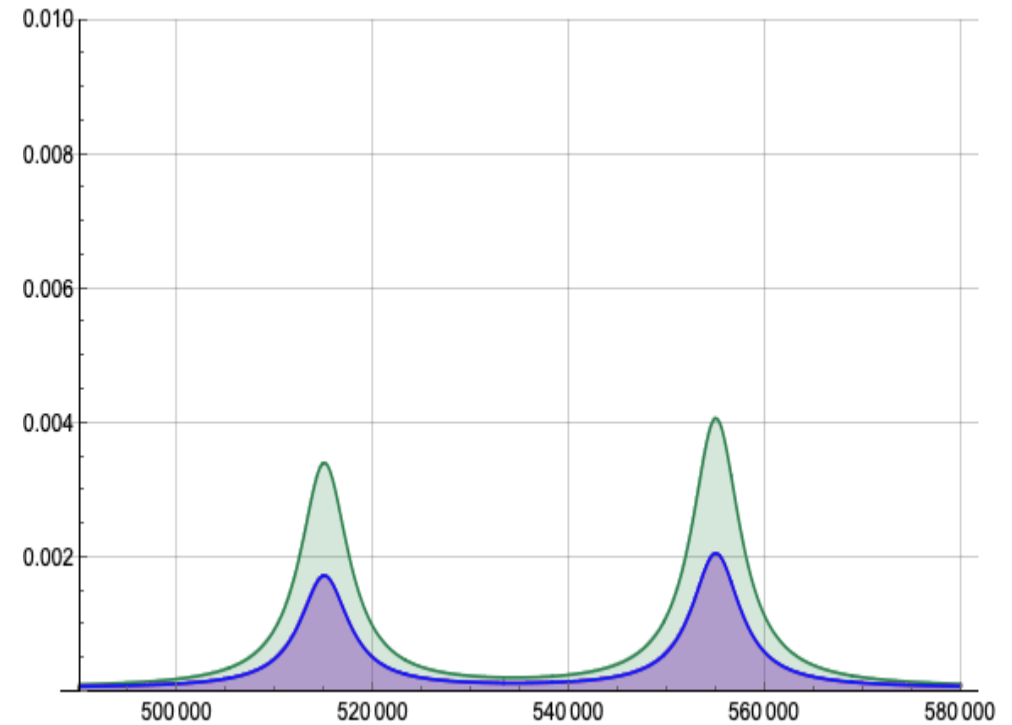
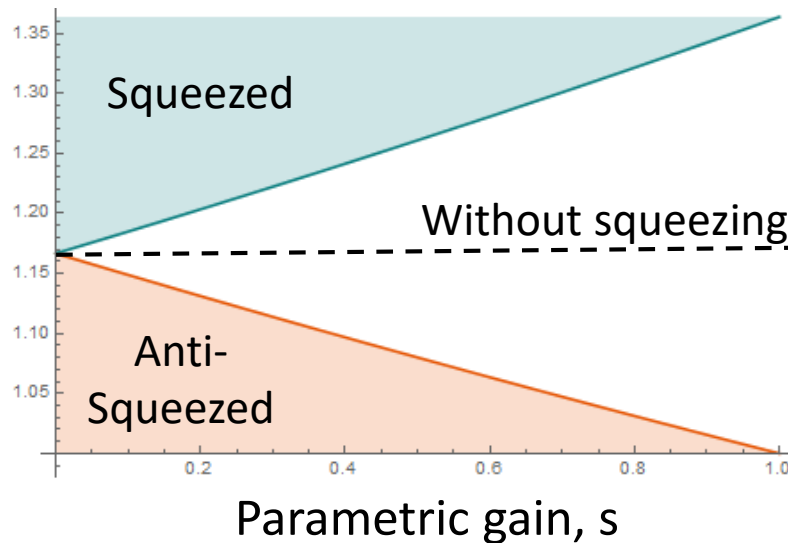


Parametric cooling close to the quantum regime

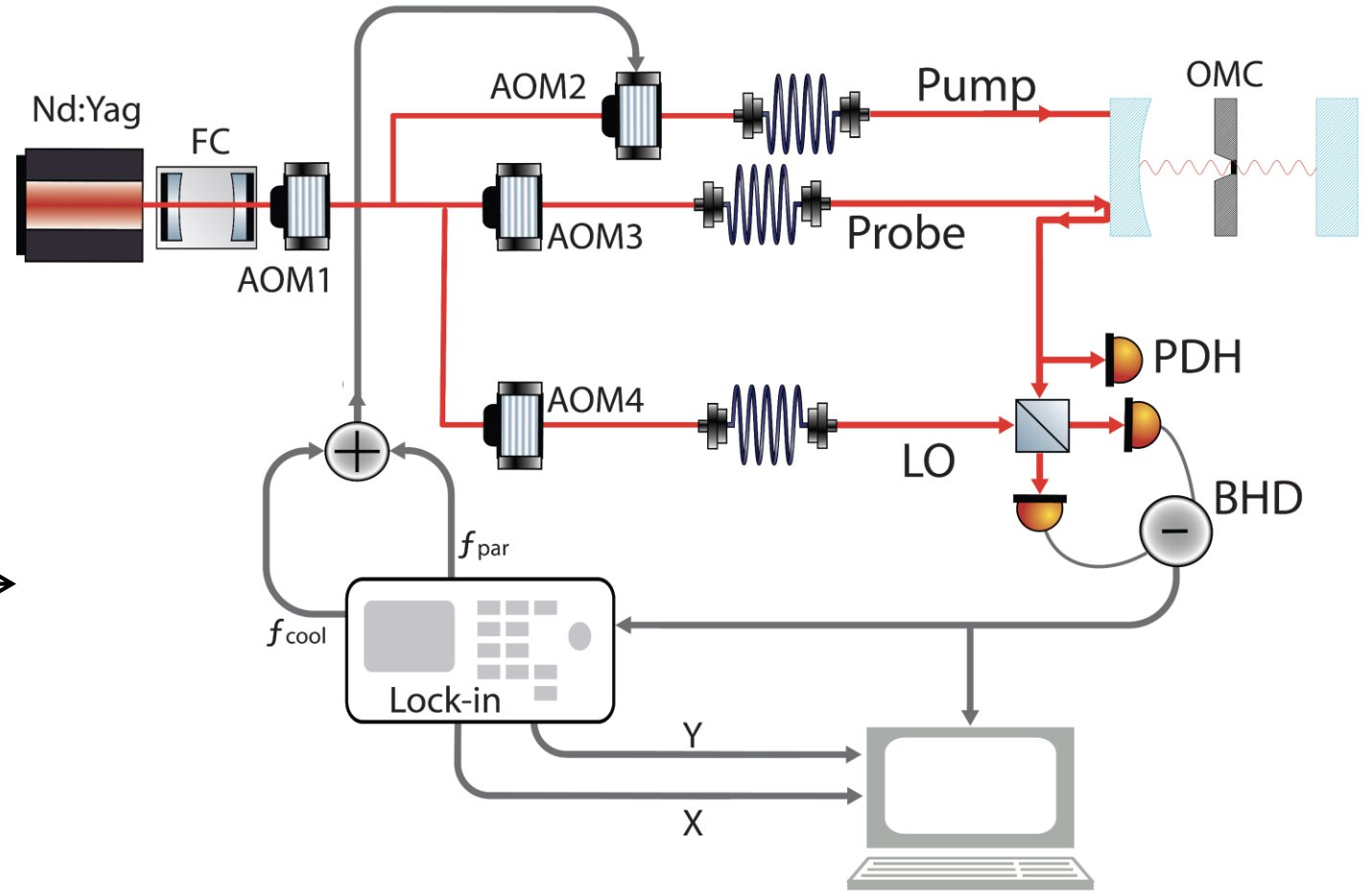
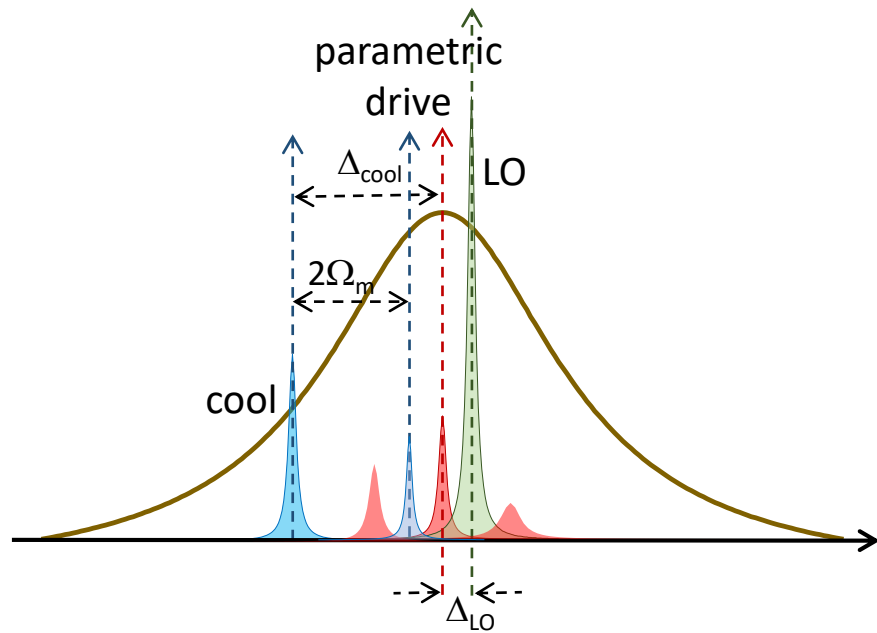


SB ratio, $R_Y : \frac{n+1+s/2}{n-s/2}$

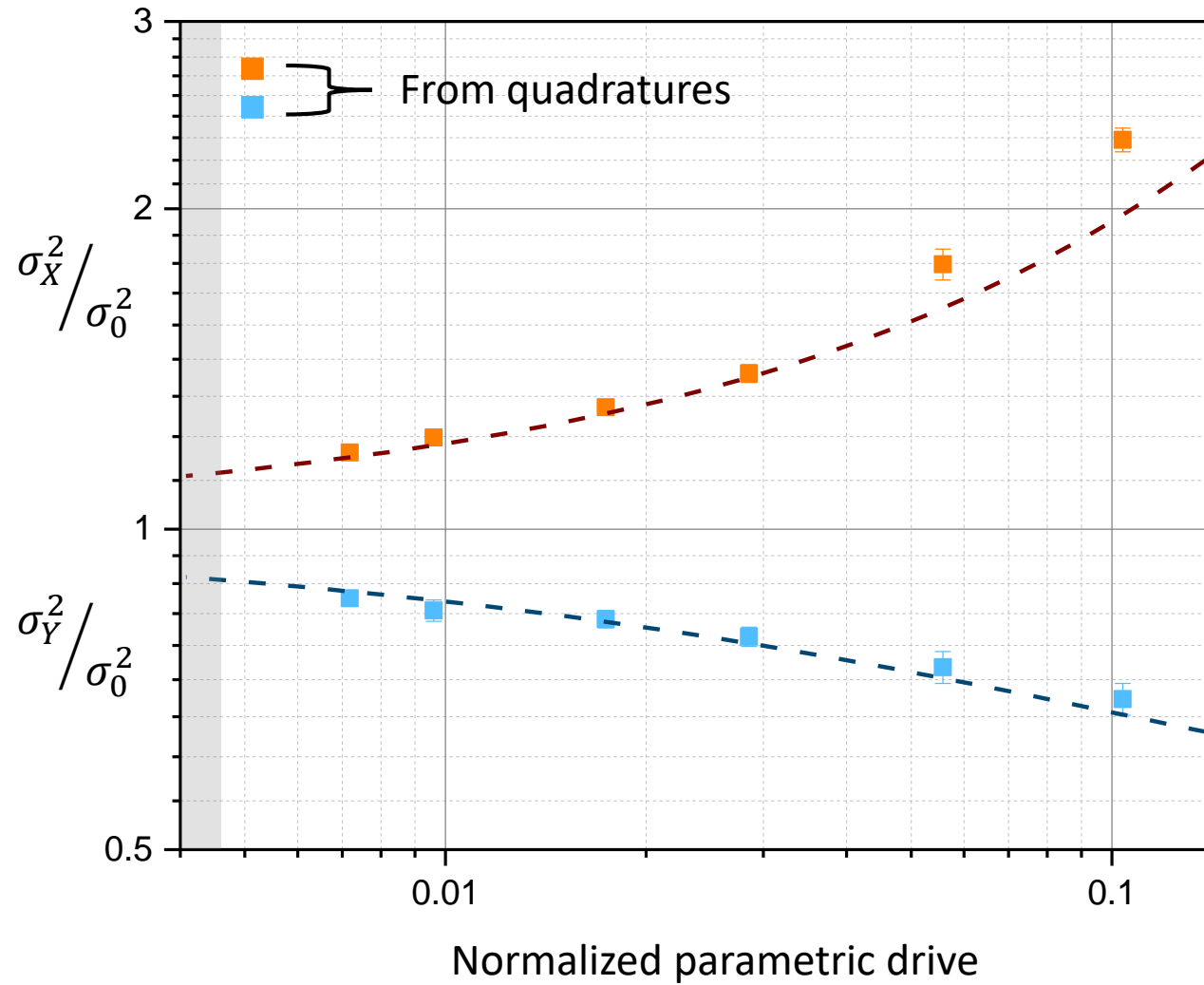
SB ratio, $R_X : \frac{n+1-s/2}{n+s/2}$



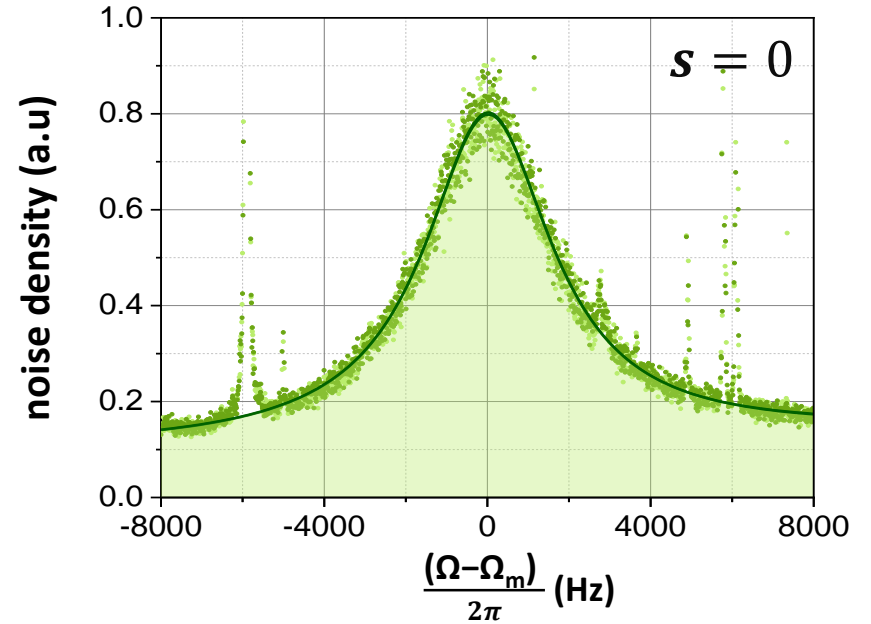
Parametric cooling: scheme



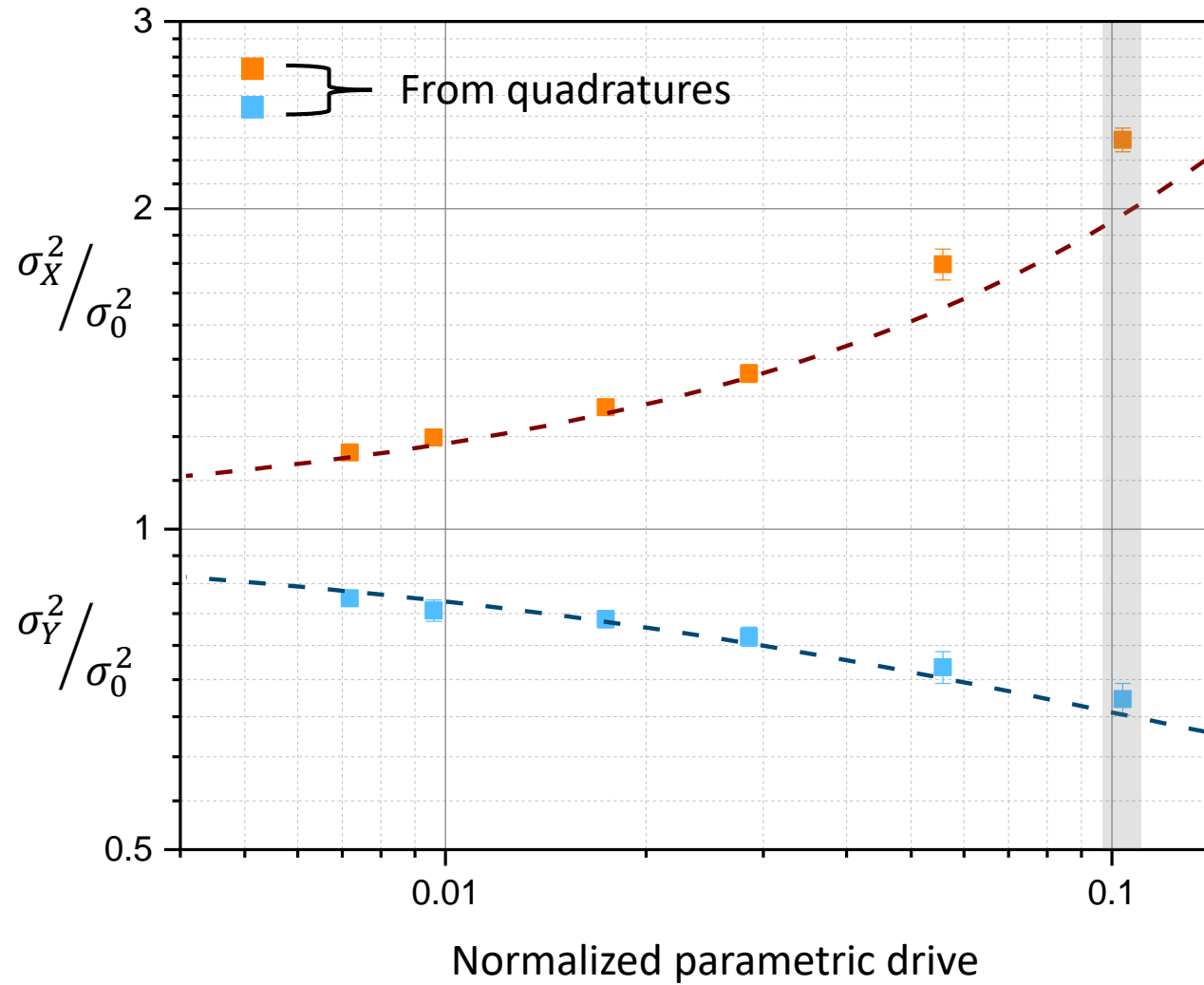
Expected variance of the quadratures



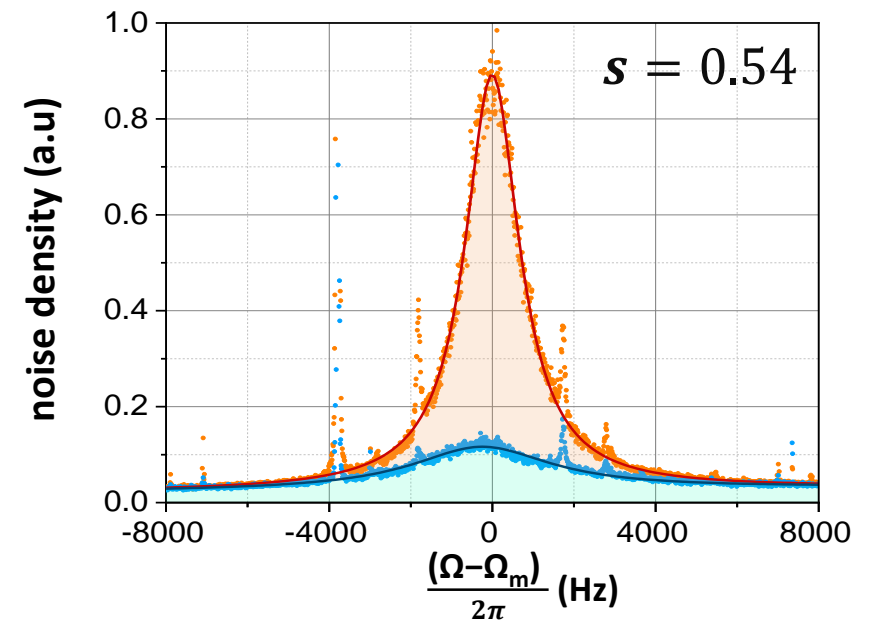
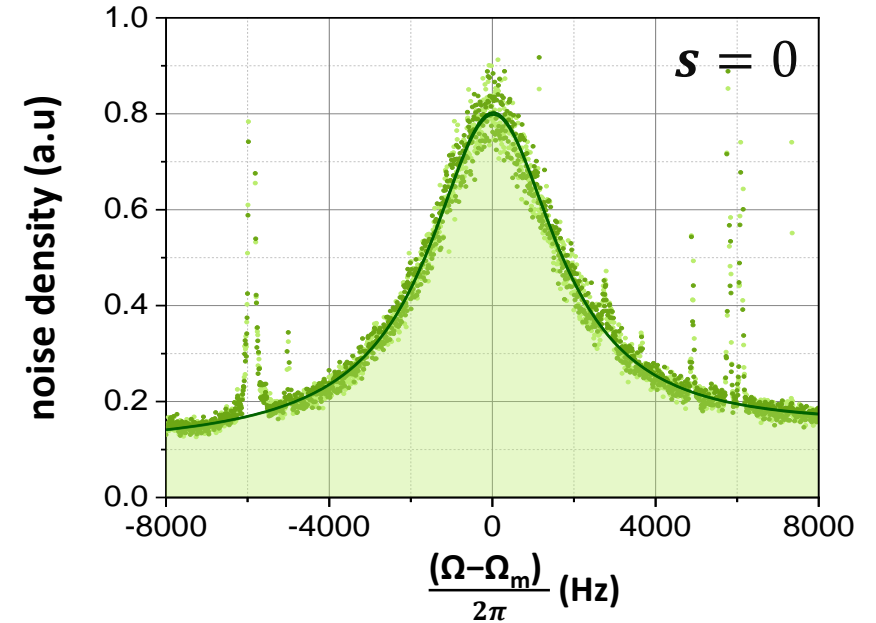
$$\sigma_X^2 = \frac{\sigma_0^2}{1-s} \quad \sigma_Y^2 = \frac{\sigma_0^2}{1+s}$$



Expected variance of the quadratures



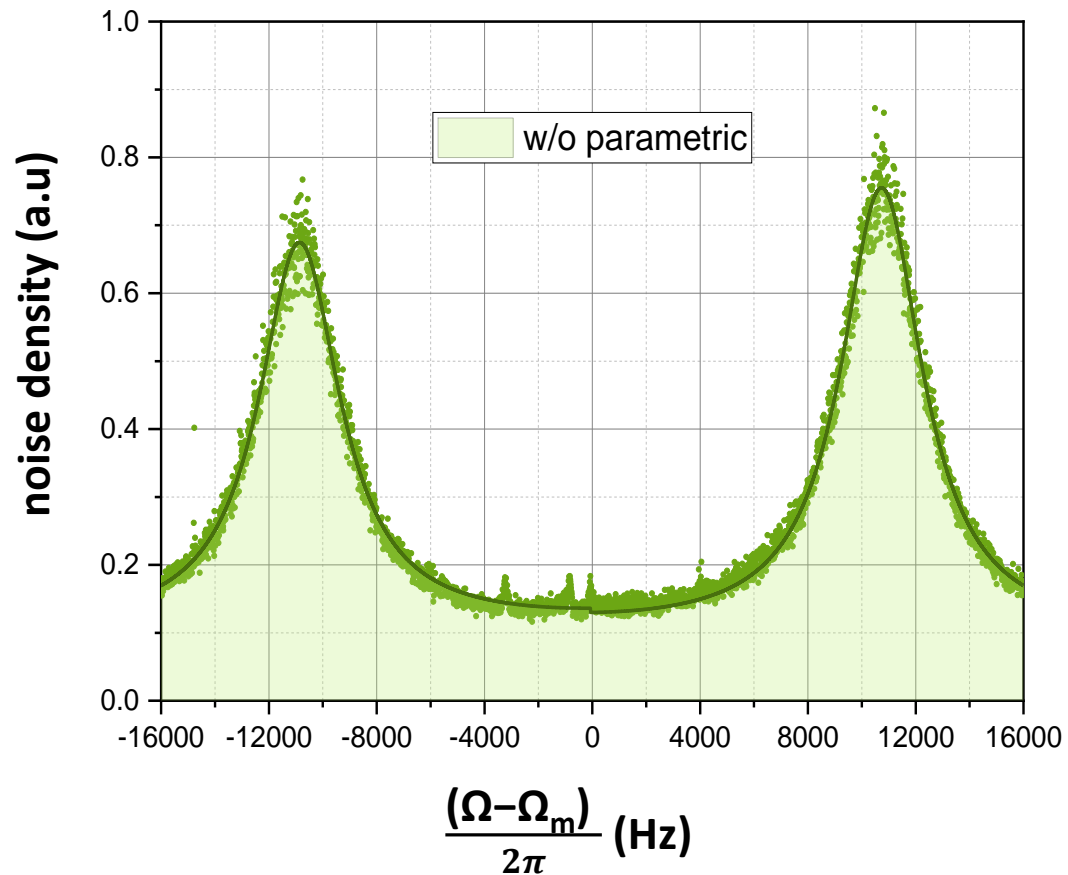
$$\sigma_X^2 = \frac{\sigma_0^2}{1-s} \quad \sigma_Y^2 = \frac{\sigma_0^2}{1+s}$$



Variation of asymmetry of the quadratures

$$S_{tot} = \frac{\Gamma_{eff}}{2} \left[\frac{A_{left}}{(\omega - \Omega_{LO})^2 + \left(\frac{\Gamma_{eff}}{2}\right)^2} + \frac{A_{right}}{(\omega + \Omega_{LO})^2 + \left(\frac{\Gamma_{eff}}{2}\right)^2} \right]$$

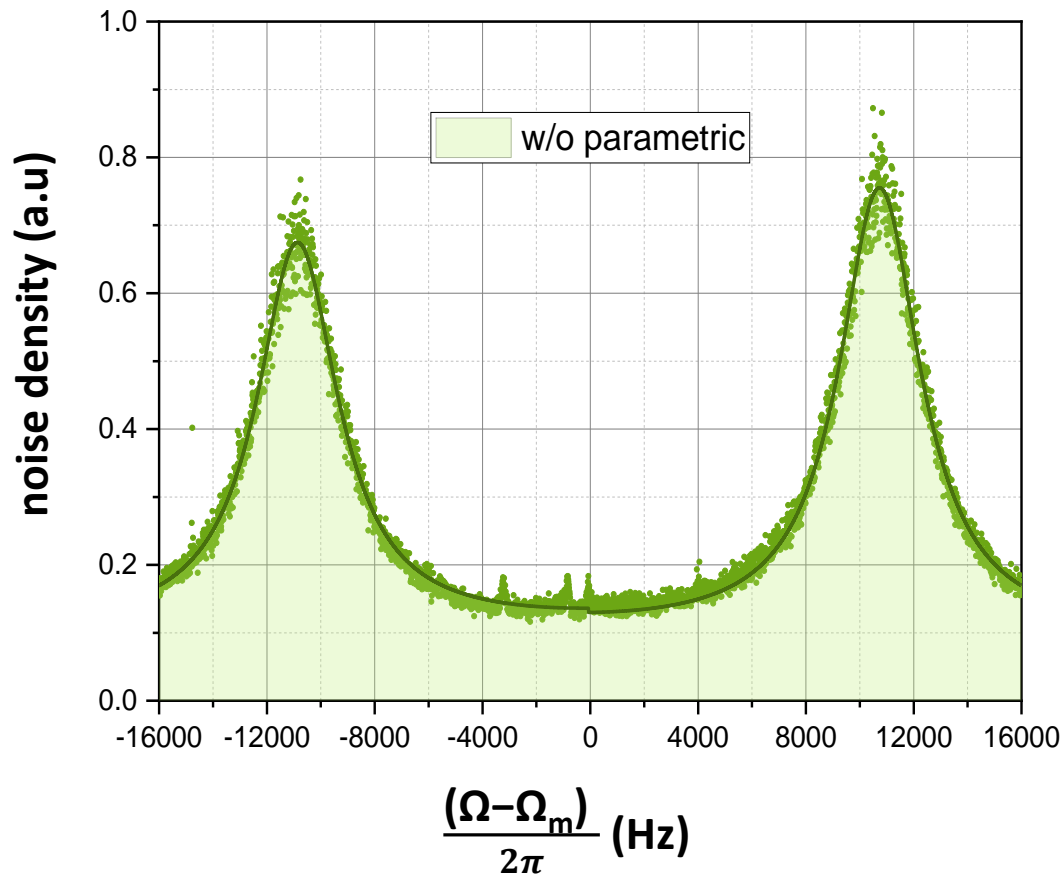
$$s = 0$$



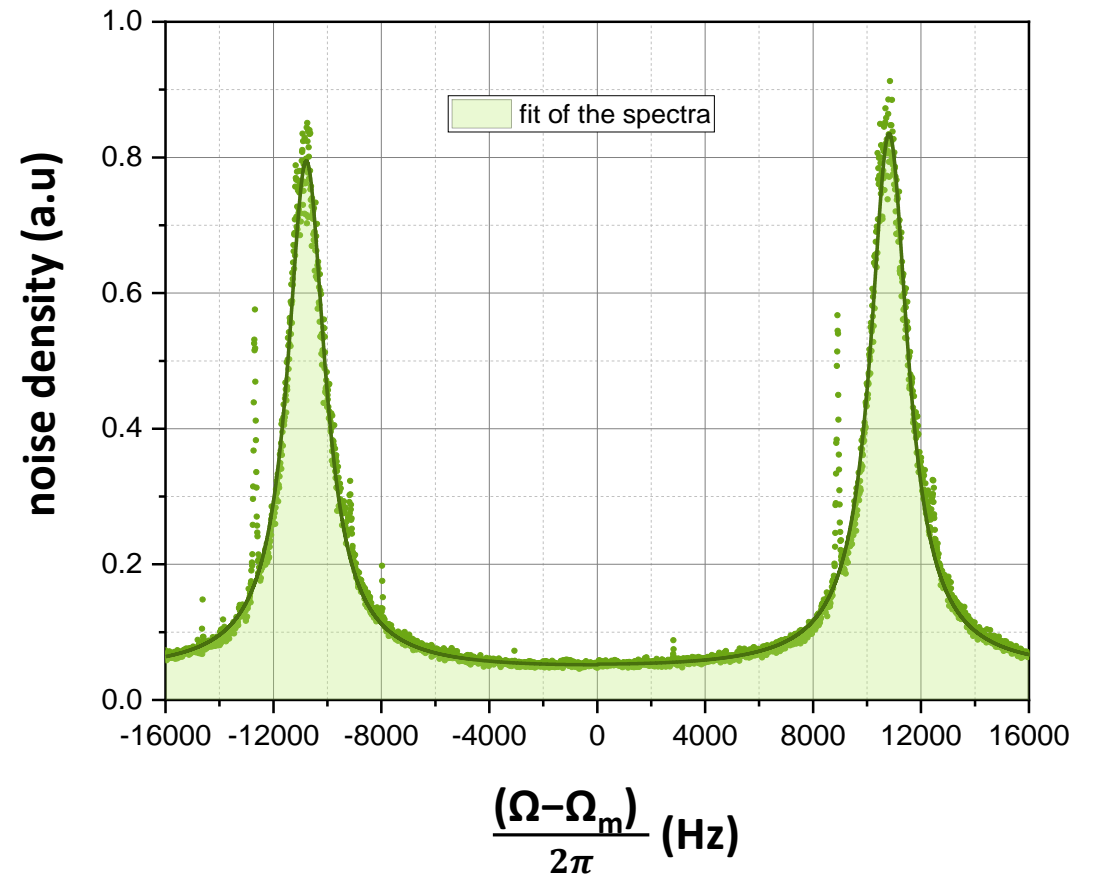
Variation of asymmetry of the quadratures

$$S_{tot} = \left[\frac{\Gamma_X A_{left}^X}{(\omega - \Omega_{LO})^2 + \left(\frac{\Gamma_X}{2}\right)^2} + \frac{\Gamma_Y A_{left}^Y}{(\omega - \Omega_{LO})^2 + \left(\frac{\Gamma_Y}{2}\right)^2} \right] + \left[\frac{\Gamma_X A_{right}^X}{(\omega + \Omega_{LO})^2 + \left(\frac{\Gamma_X}{2}\right)^2} + \frac{\Gamma_Y A_{right}^Y}{(\omega + \Omega_{LO})^2 + \left(\frac{\Gamma_Y}{2}\right)^2} \right]$$

$s = 0$



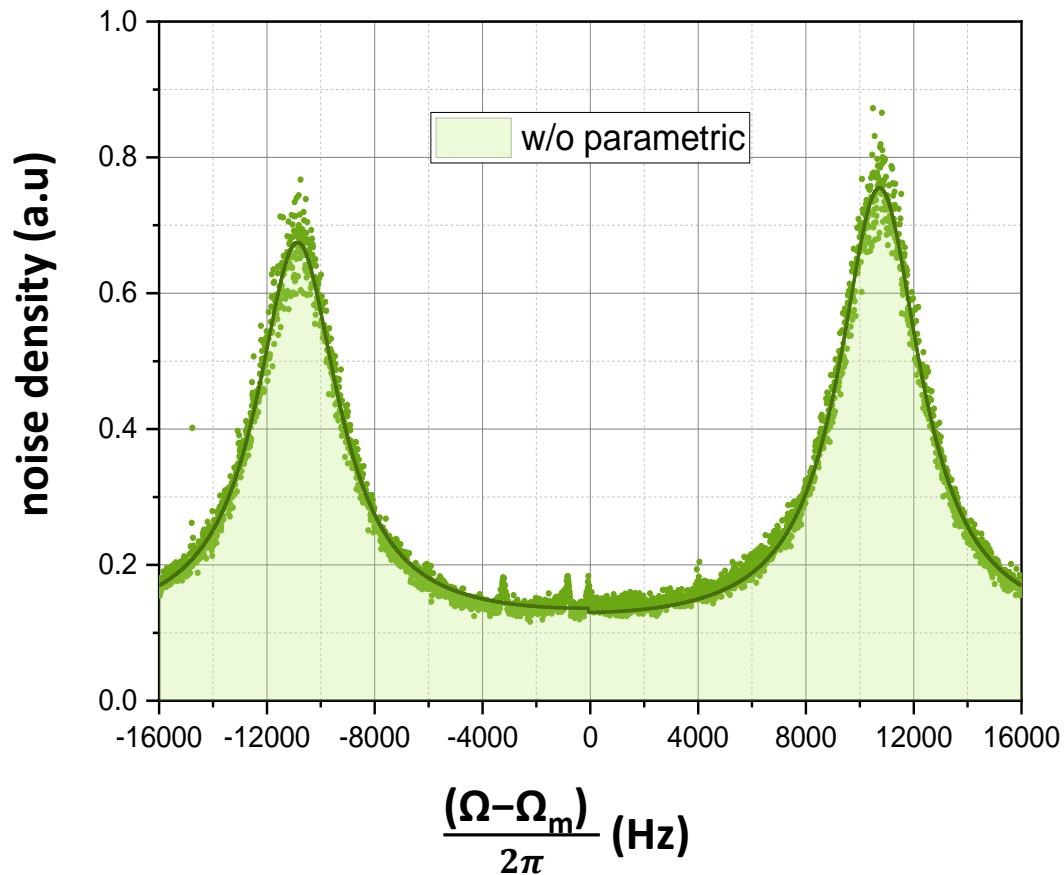
$s = 0.54$



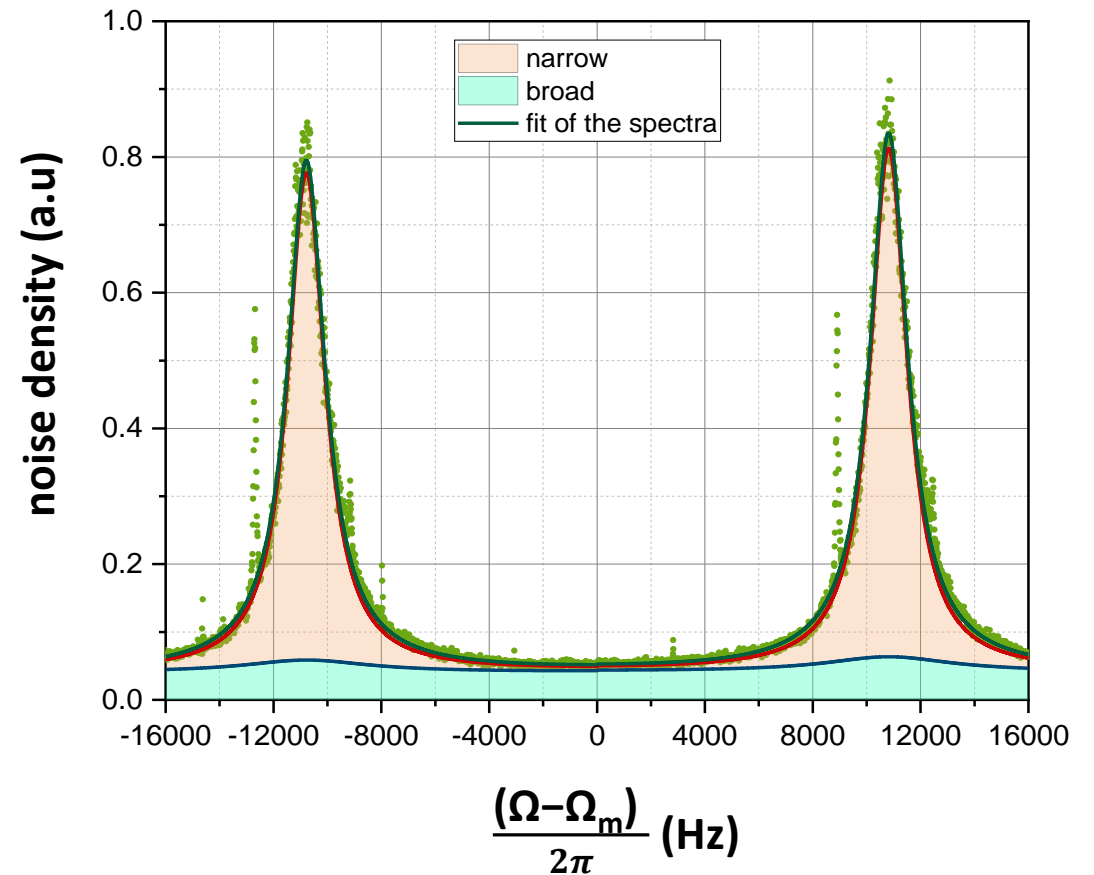
Variation of asymmetry of the quadratures

$$S_{tot} = \frac{\Gamma_X A_{left}^X}{(\omega - \Omega_{LO})^2 + \left(\frac{\Gamma_X}{2}\right)^2} + \frac{\Gamma_Y A_{left}^Y}{(\omega - \Omega_{LO})^2 + \left(\frac{\Gamma_Y}{2}\right)^2} + \frac{\Gamma_X A_{right}^X}{(\omega + \Omega_{LO})^2 + \left(\frac{\Gamma_X}{2}\right)^2} + \frac{\Gamma_Y A_{right}^Y}{(\omega + \Omega_{LO})^2 + \left(\frac{\Gamma_Y}{2}\right)^2}$$

$s = 0$



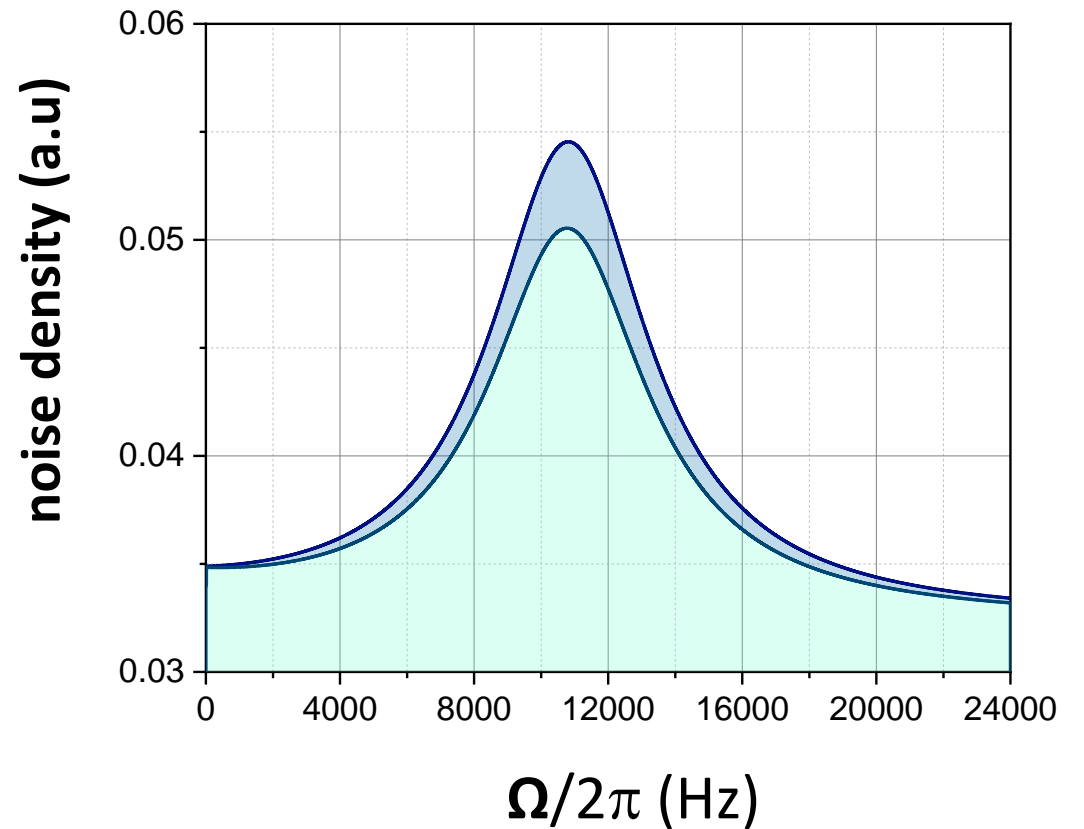
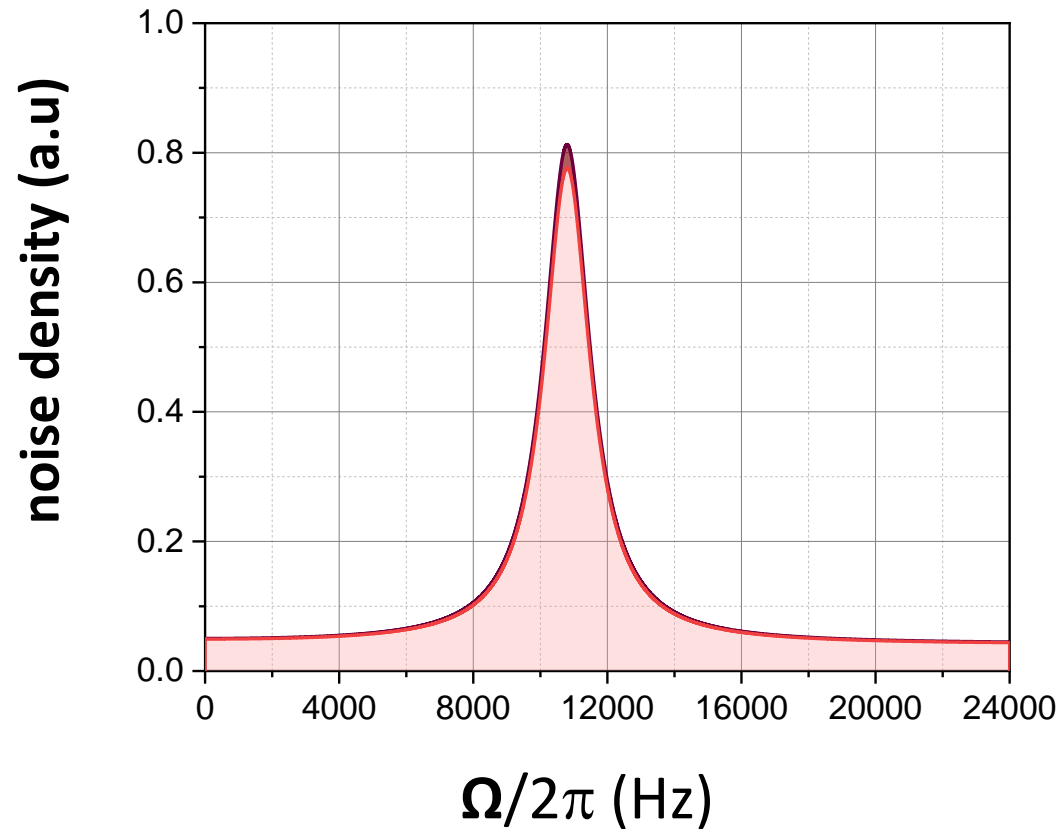
$s = 0.54$



Variation of asymmetry of the quadratures

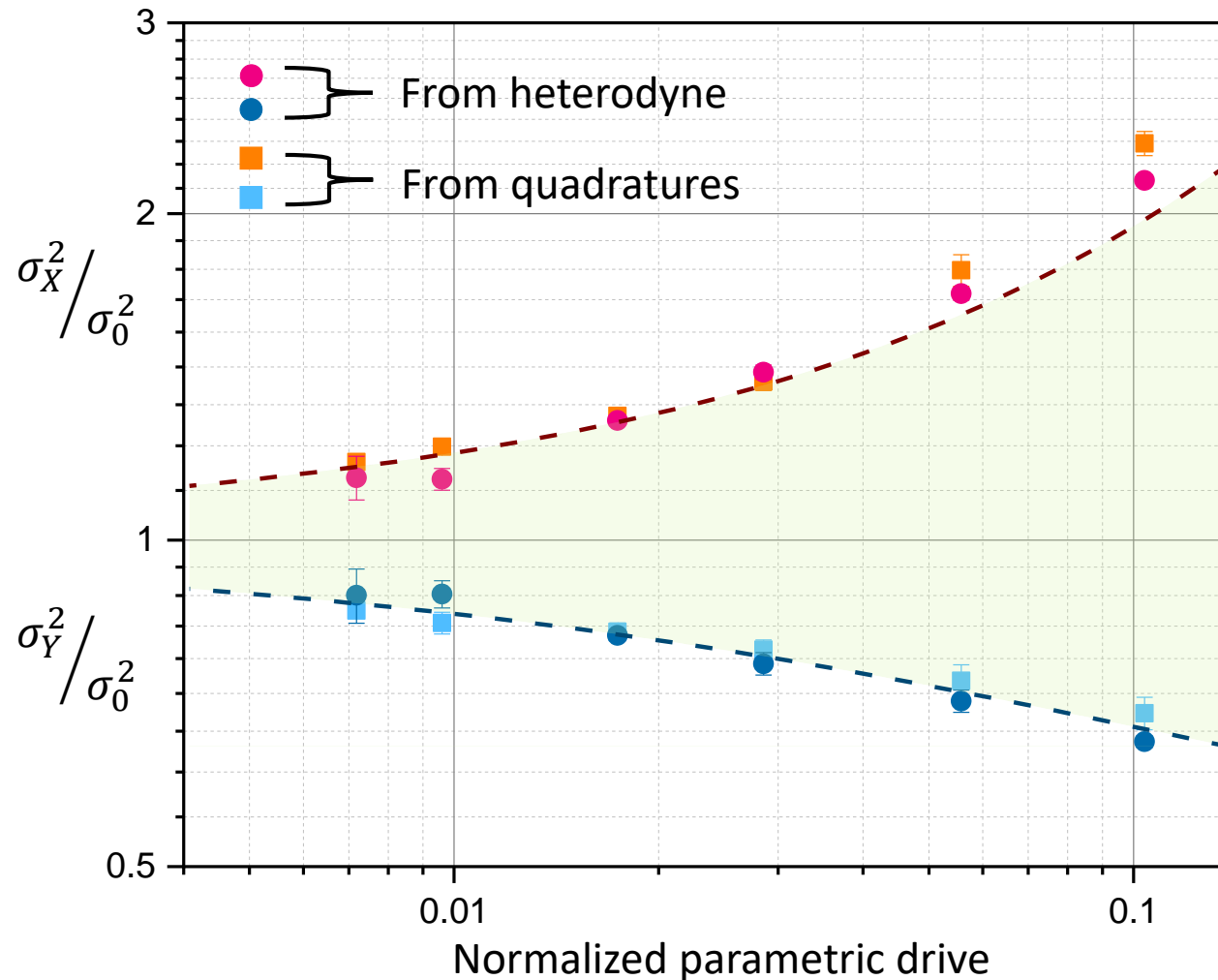
$$S_{tot} = \frac{\Gamma_X A_{left}^X}{(\omega - \Omega_{LO})^2 + \left(\frac{\Gamma_X}{2}\right)^2} + \frac{\Gamma_Y A_{left}^Y}{(\omega - \Omega_{LO})^2 + \left(\frac{\Gamma_Y}{2}\right)^2} + \frac{\Gamma_X A_{right}^X}{(\omega + \Omega_{LO})^2 + \left(\frac{\Gamma_X}{2}\right)^2} + \frac{\Gamma_Y A_{right}^Y}{(\omega + \Omega_{LO})^2 + \left(\frac{\Gamma_Y}{2}\right)^2}$$

$$s = 0.54$$



Variation of asymmetry of the quadratures

$$S_{tot} = \frac{\Gamma_X A_{left}^X}{(\omega - \Omega_{LO})^2 + \left(\frac{\Gamma_X}{2}\right)^2} + \frac{\Gamma_Y A_{left}^Y}{(\omega - \Omega_{LO})^2 + \left(\frac{\Gamma_Y}{2}\right)^2} + \frac{\Gamma_X A_{right}^X}{(\omega + \Omega_{LO})^2 + \left(\frac{\Gamma_X}{2}\right)^2} + \frac{\Gamma_Y A_{right}^Y}{(\omega + \Omega_{LO})^2 + \left(\frac{\Gamma_Y}{2}\right)^2}$$



$$\Gamma_X = \Gamma_{eff}(1 - s)$$

$$\Gamma_Y = \Gamma_{eff}(1 + s)$$

$$\sigma_X^2 = \frac{\sigma_0^2}{1 - s}$$

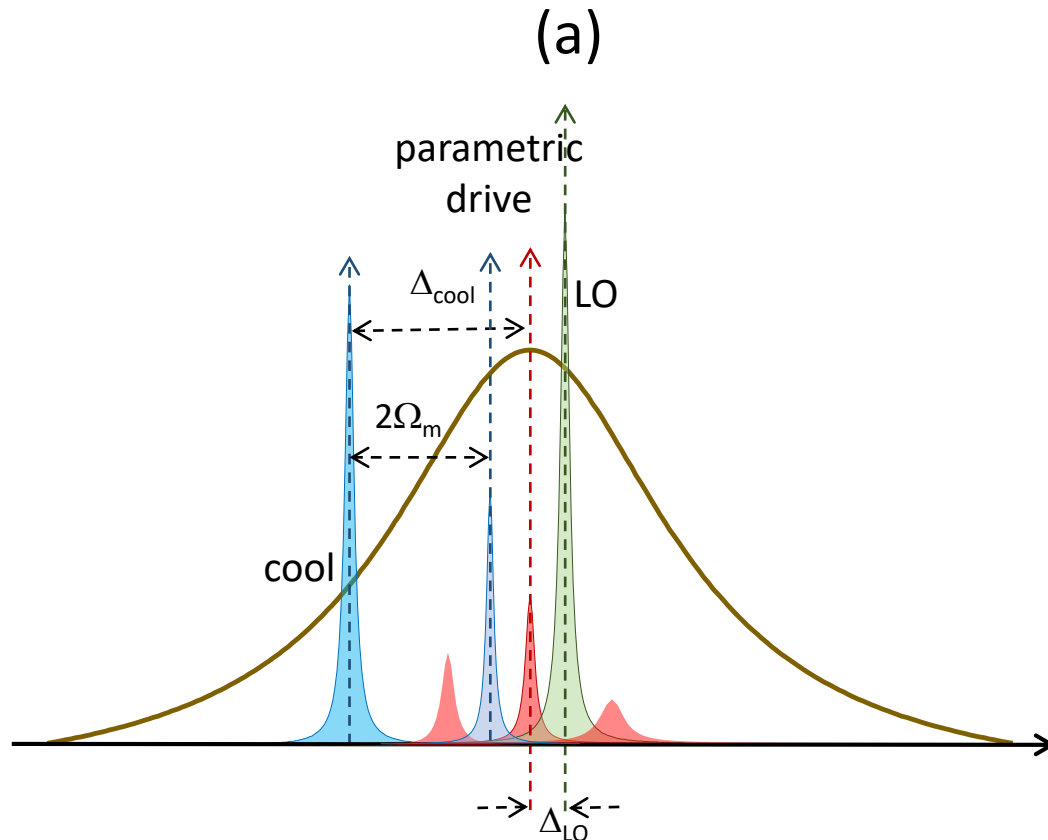
$$\sigma_Y^2 = \frac{\sigma_0^2}{1 + s}$$

Variation of asymmetry of the quadratures

$$I_{pump} = I_{cool} + I_{par}, \text{ where: } I_{par} = \alpha I_{pump}$$

(a) $I_{pump} \uparrow$ **keeping 'α' constant: 's' is constant**

(b) $I_{par} \uparrow$ **keeping I_{pump} constant: 's' varies keeping Γ_{eff} constant**

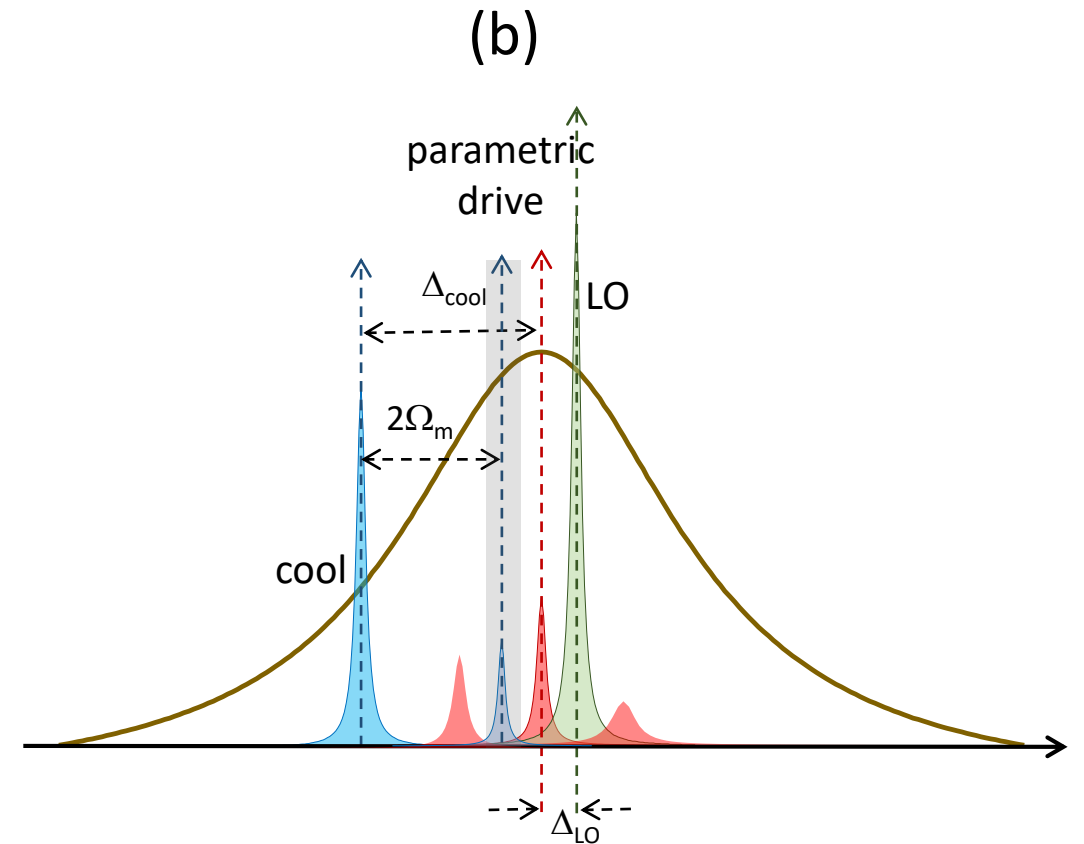
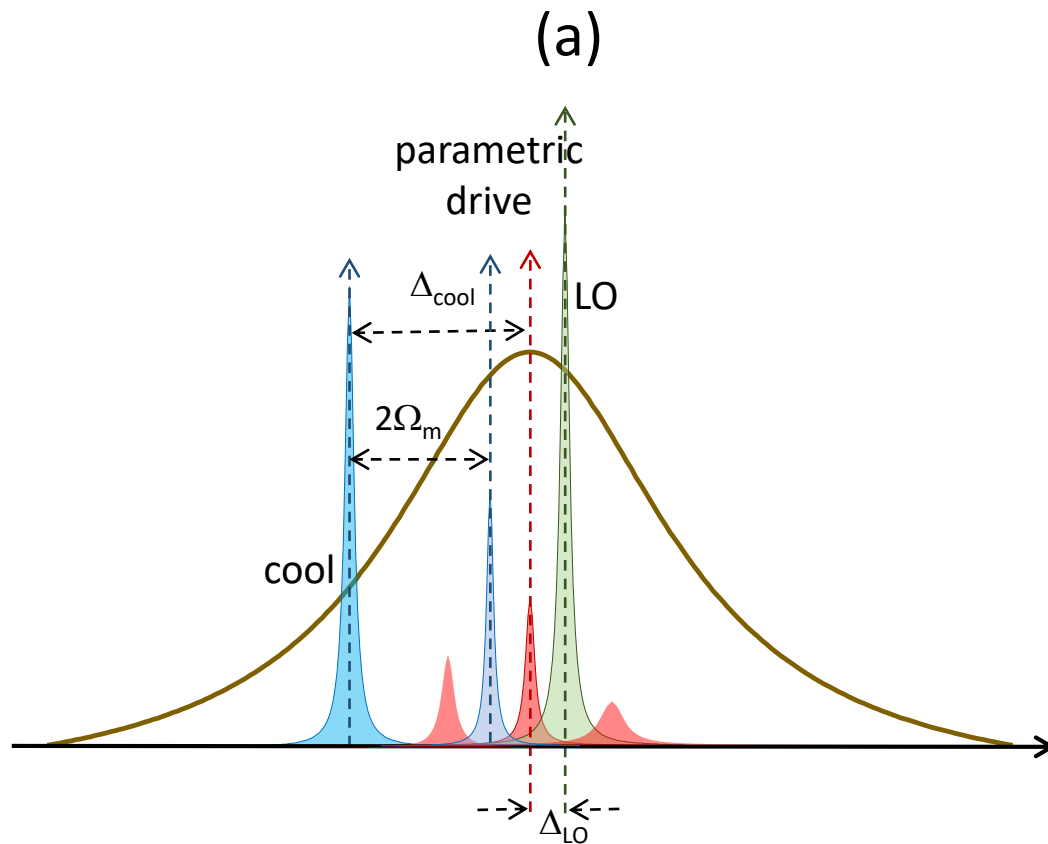


Variation of asymmetry of the quadratures

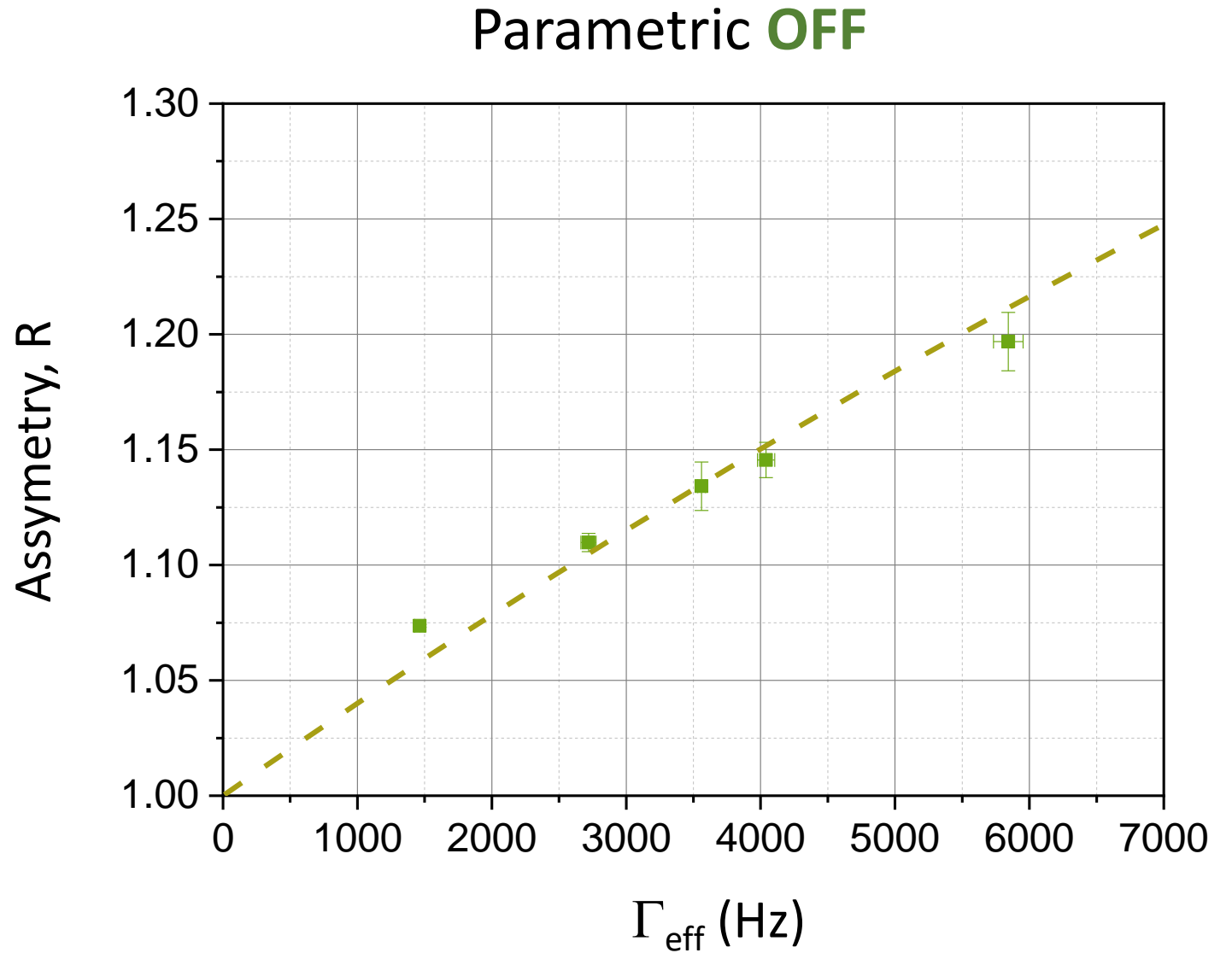
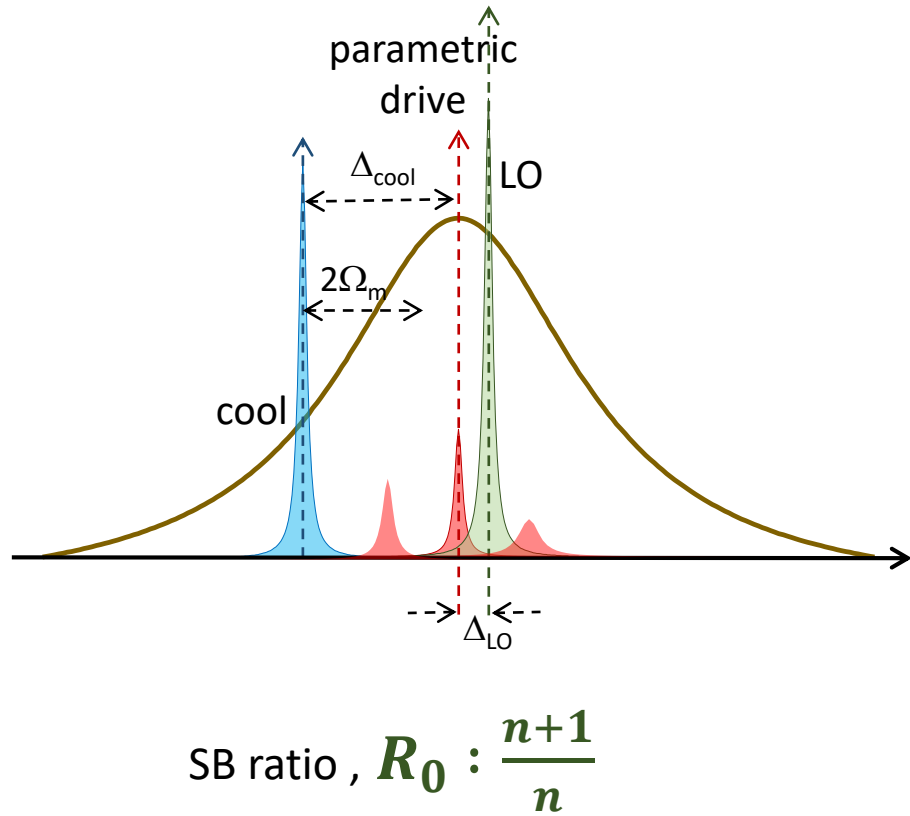
$$I_{pump} = I_{cool} + I_{par}, \text{ where: } I_{par} = \alpha I_{pump}$$

(a) $I_{pump} \uparrow$ keeping ' α ' constant: ' s ' is constant

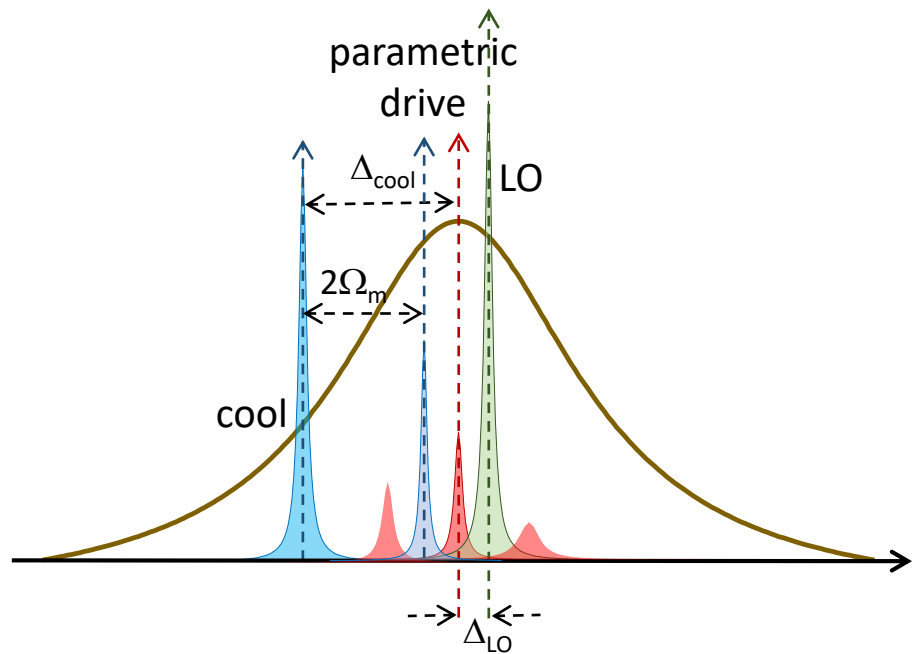
(b) $I_{par} \uparrow$ keeping I_{pump} constant: ' s ' varies keeping Γ_{eff} constant



Asymmetry with pump power



Asymmetry with pump power



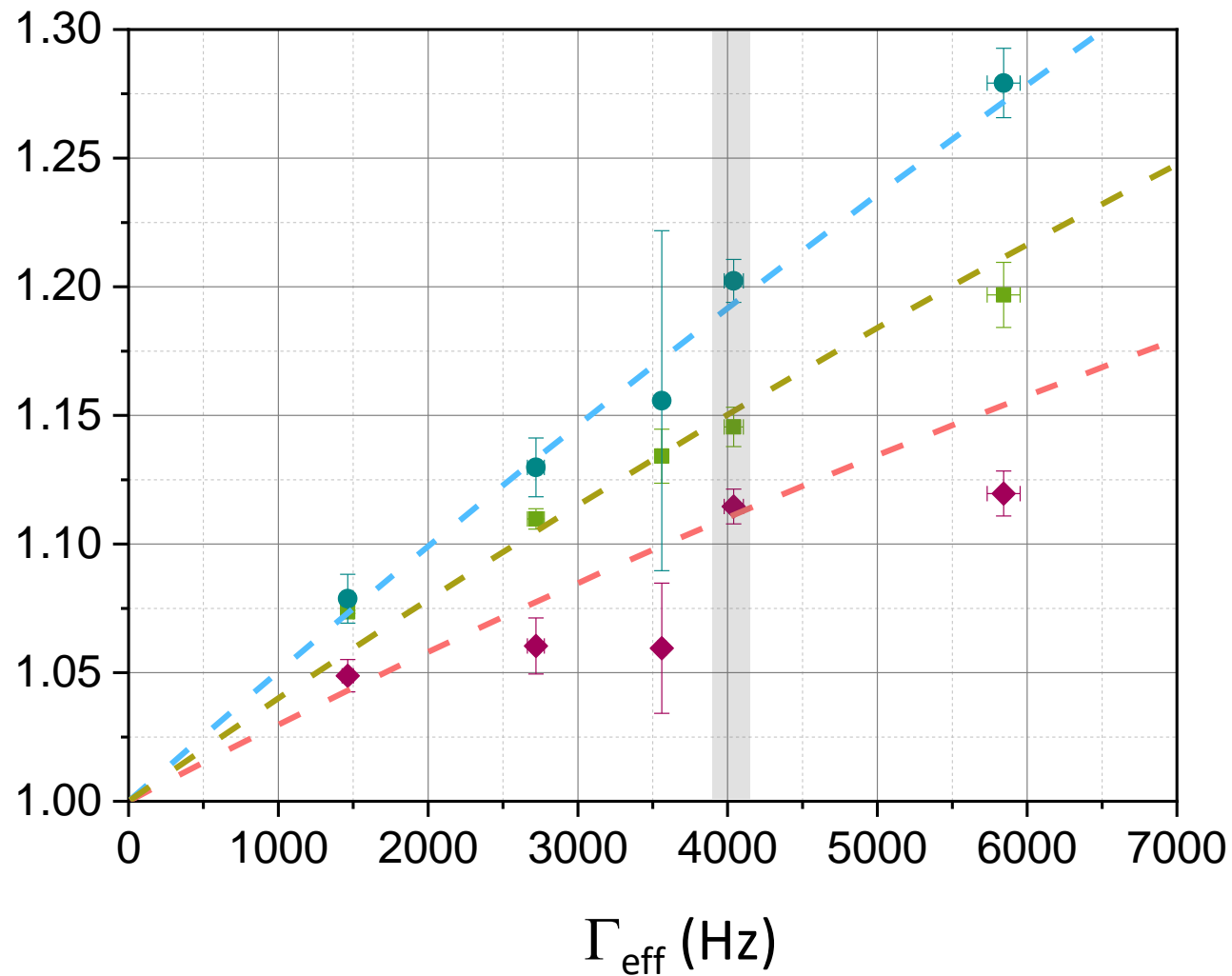
$$\text{SB ratio, } R_0 : \frac{n+1}{n}$$

$$\text{SB ratio, } R_+ : \frac{n+1+s/2}{n-s/2}$$

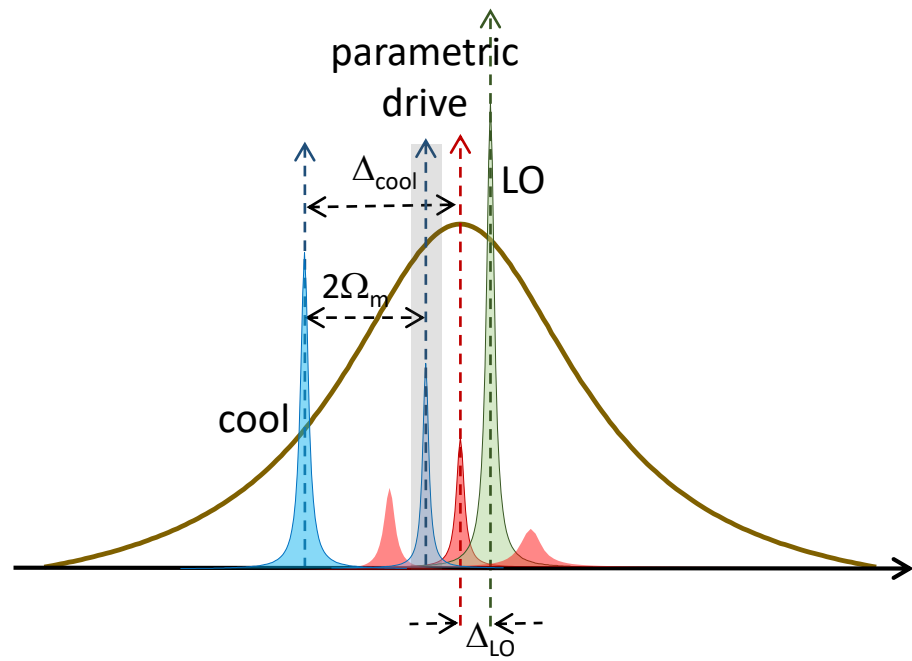
$$\text{SB ratio, } R_- : \frac{n+1-s/2}{n+s/2}$$

Assymetry, R

Parametric **ON**



Asymmetry as a function of 's'

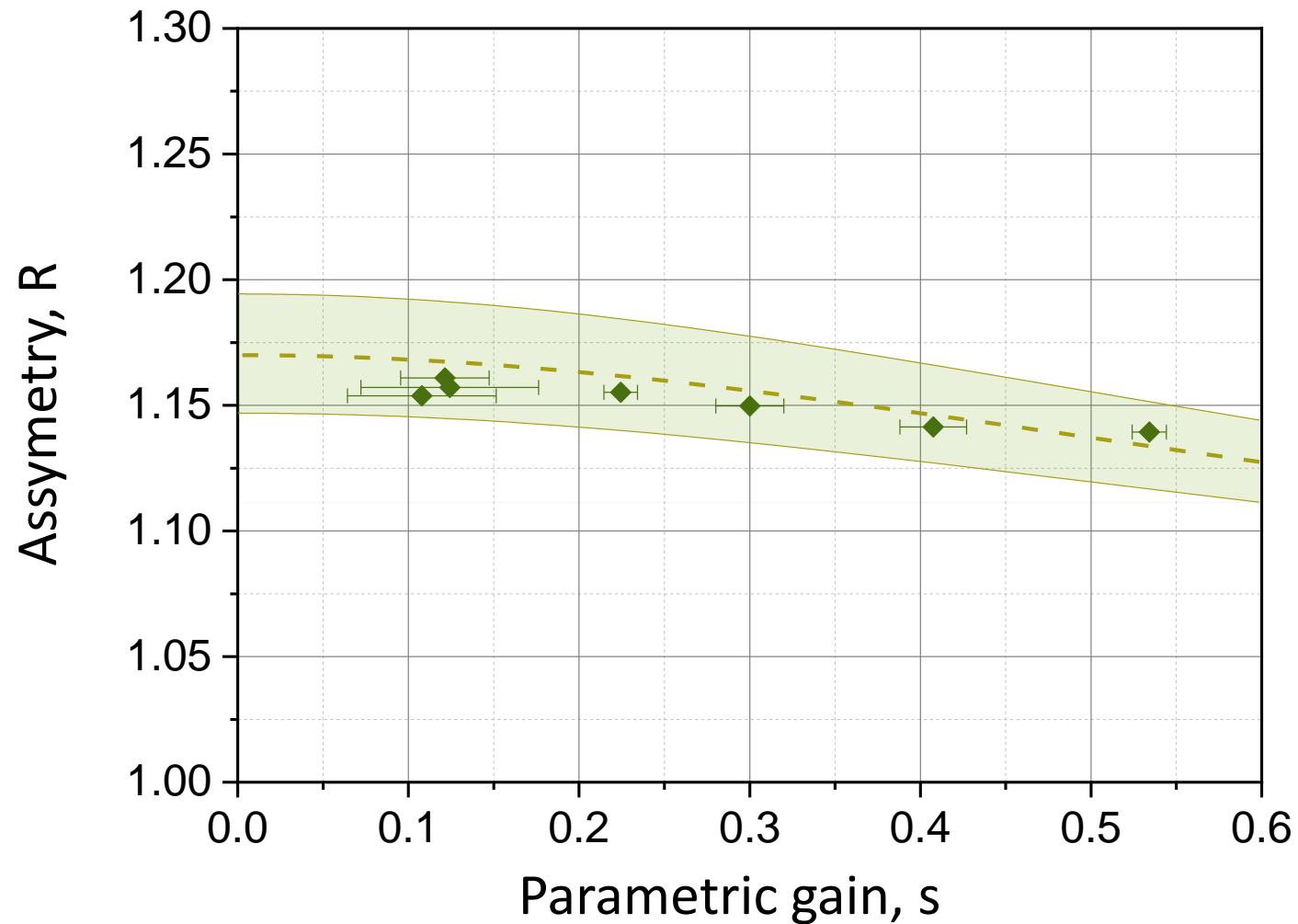


SB ratio, $R_0 : \frac{n+1}{n}$

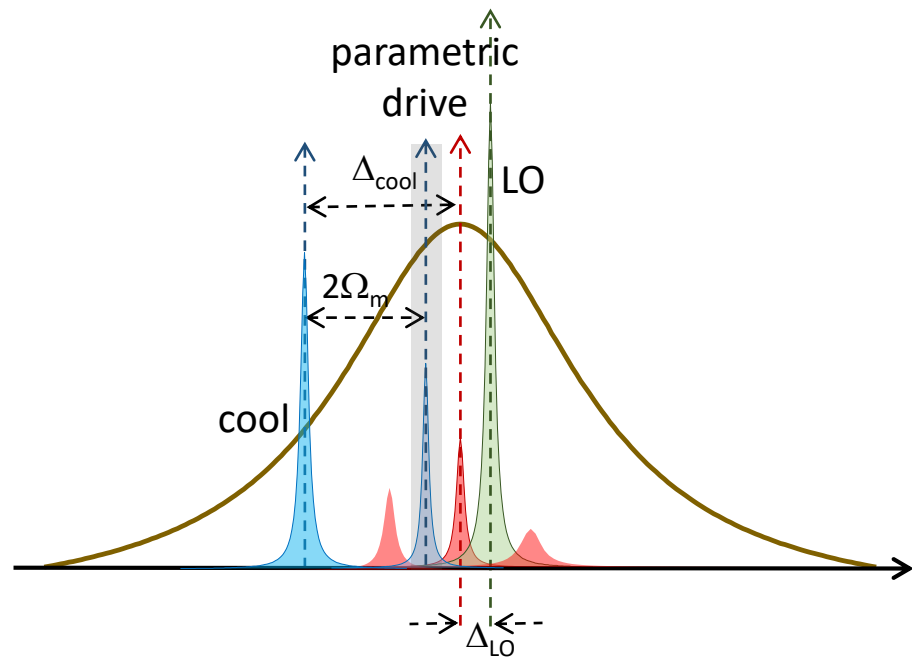
SB ratio, $R_+ : \frac{n+1+s/2}{n-s/2}$

SB ratio, $R_- : \frac{n+1-s/2}{n+s/2}$

Parametric **OFF**



Asymmetry as a function of 's'

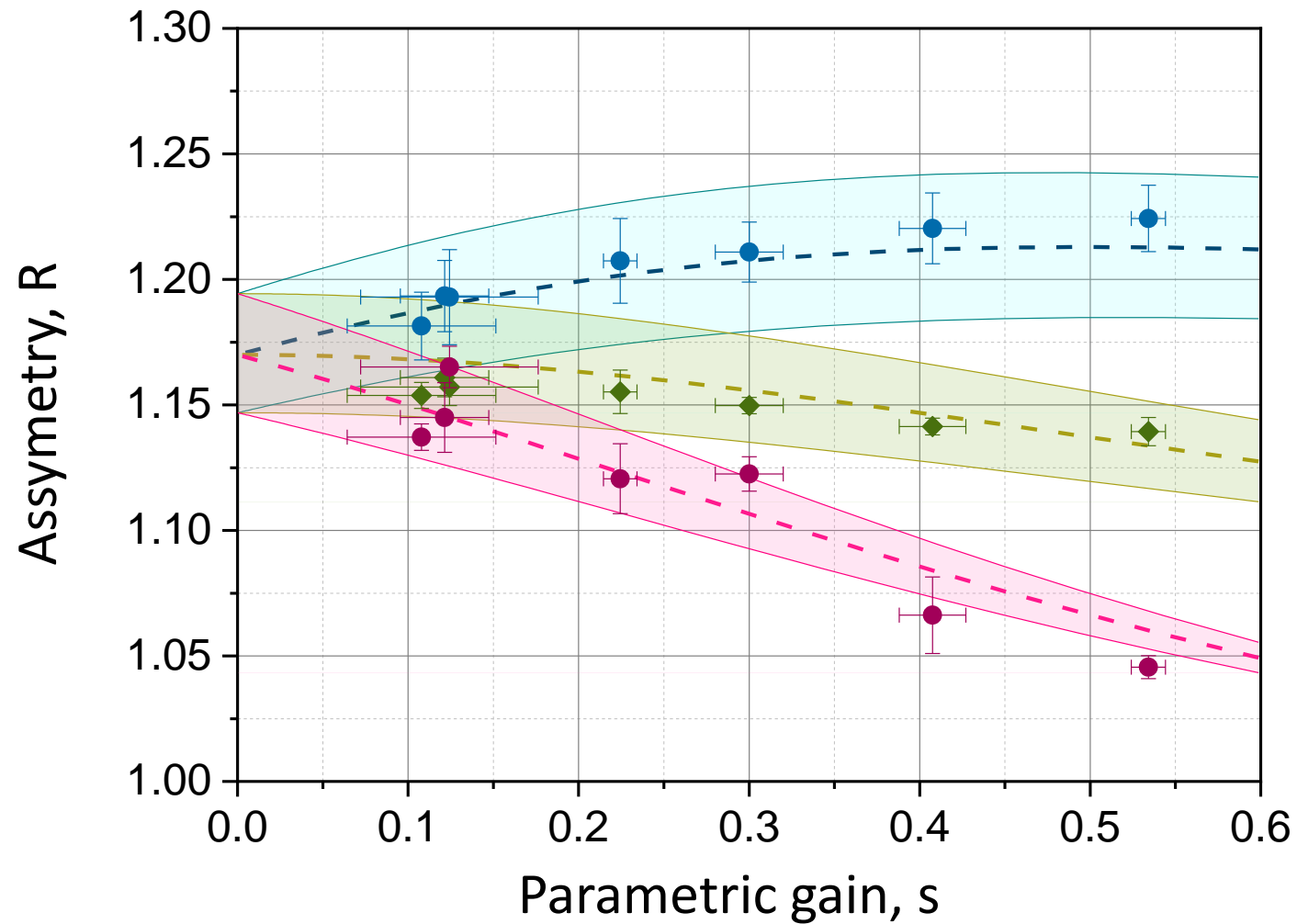


SB ratio, $R_0 : \frac{n+1}{n}$

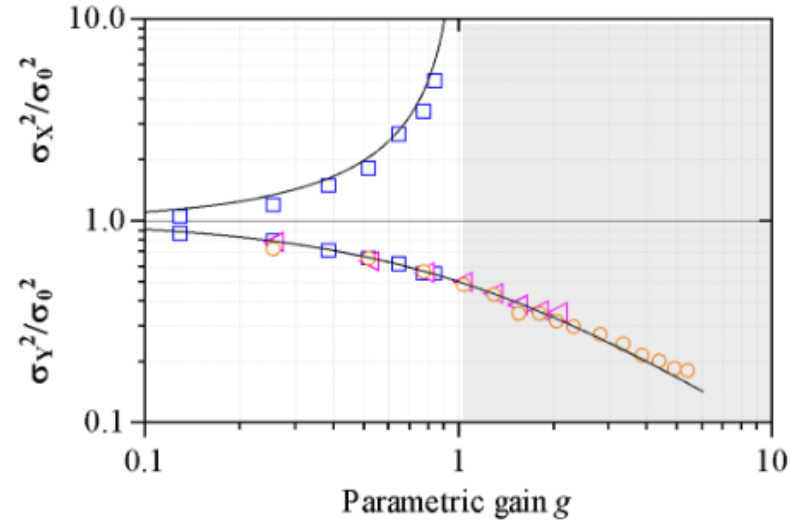
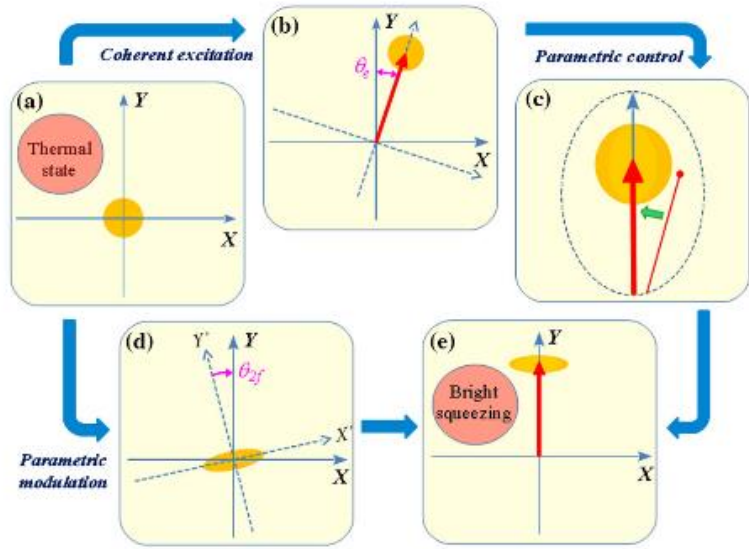
SB ratio, $R_+ : \frac{n+1+s/2}{n-s/2}$

SB ratio, $R_- : \frac{n+1-s/2}{n+s/2}$

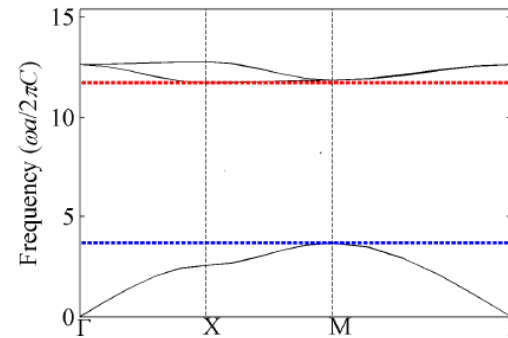
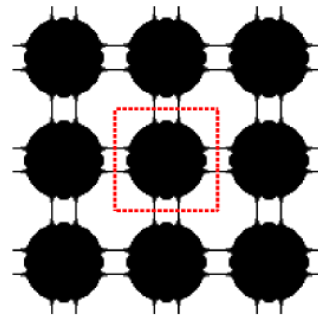
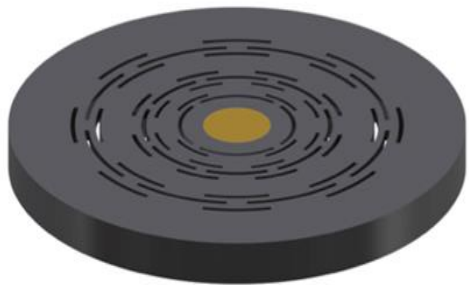
Parametric **ON**



Perspectives



Parametric feedback control to realize squeezing below 3dB limit close to quantum regime



$$Q \sim 10^8$$

$$n \sim 10$$

Quantum squeezing at room temperature

Thanks



Francesco Marin



Francesco Marino



Paolo Vezio



Massimo Calamai